Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 7 April 2011, 14:00 – 17:00 hours

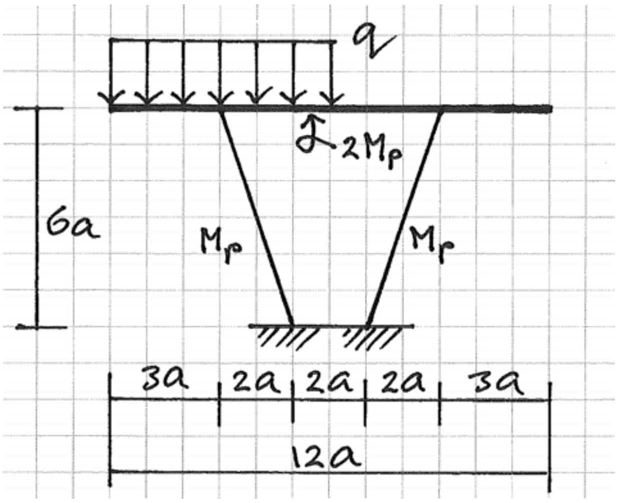

Figure 1. Frame structure

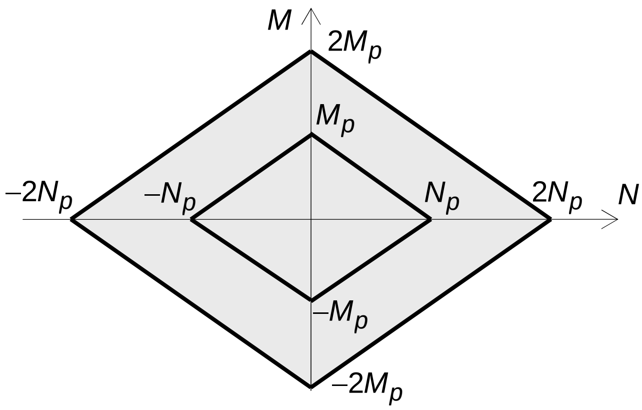

Figure 2. Yield contours

Problem 1

A frame consists of two columns and a beam (Fig. 1). The beam is two times as strong as the columns. The joints are fixed connections. The structure is loaded by a vertical load *q* that is evenly distributed over the left half of the beam. The relation of Figure 2 exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load *q* for all possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 9\sqrt{10}$. Choose one of the following problems (You need not do both).
– Use Figure 3 to determine the largest lower-bound for *q*.
– Determine the smallest upper-bound for *q*.
You only need to write down the equations and not solve the equations (1.5 points).

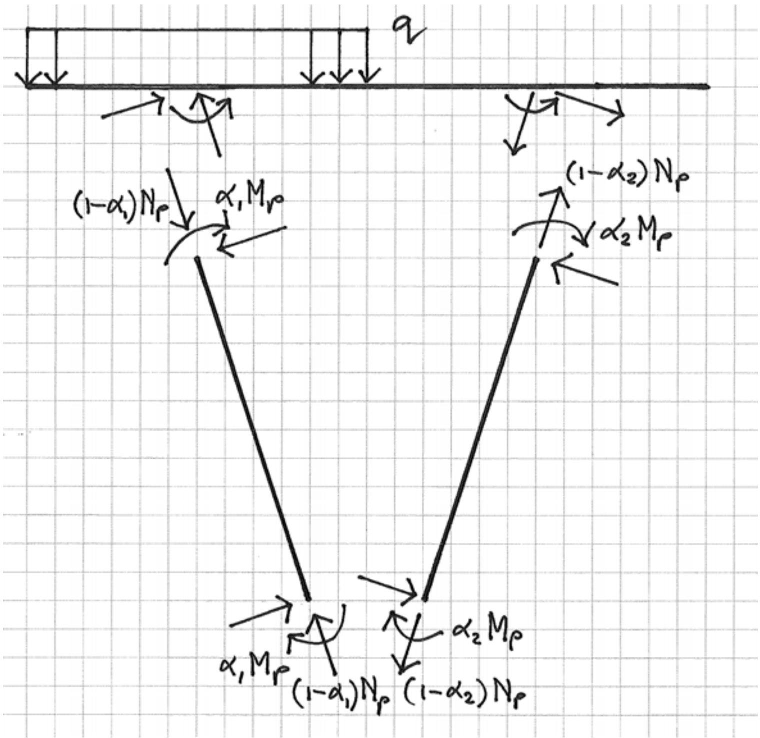

Figure 3. Equilibrium system for including M-N interaction

Problem 2

A reinforced concrete plate has two simply supported edges (Fig. 4). It carries an evenly distributed load q over half of the plate (shaded area). The plate is homogeneous and orthotropic.

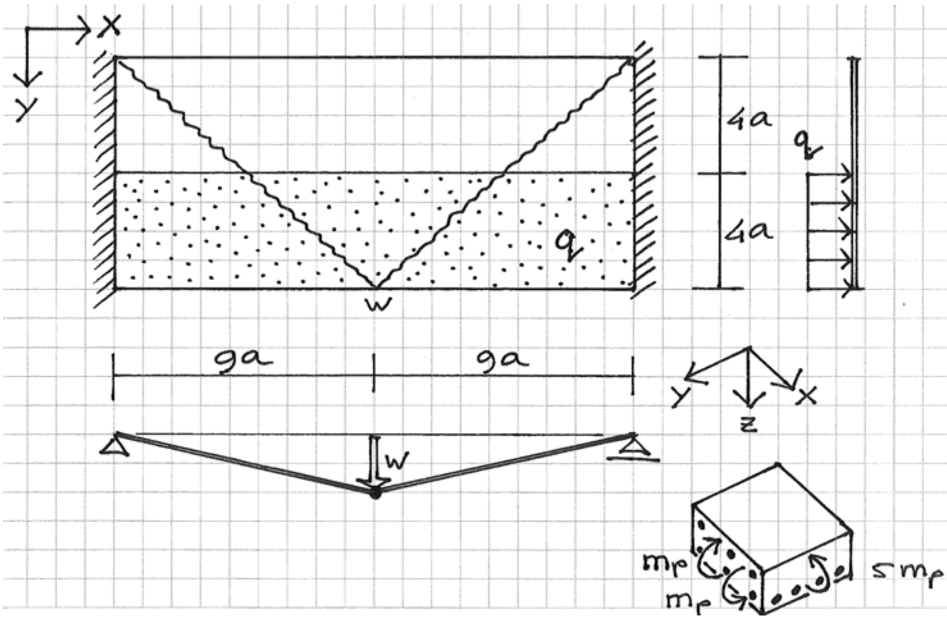

Figure 4. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

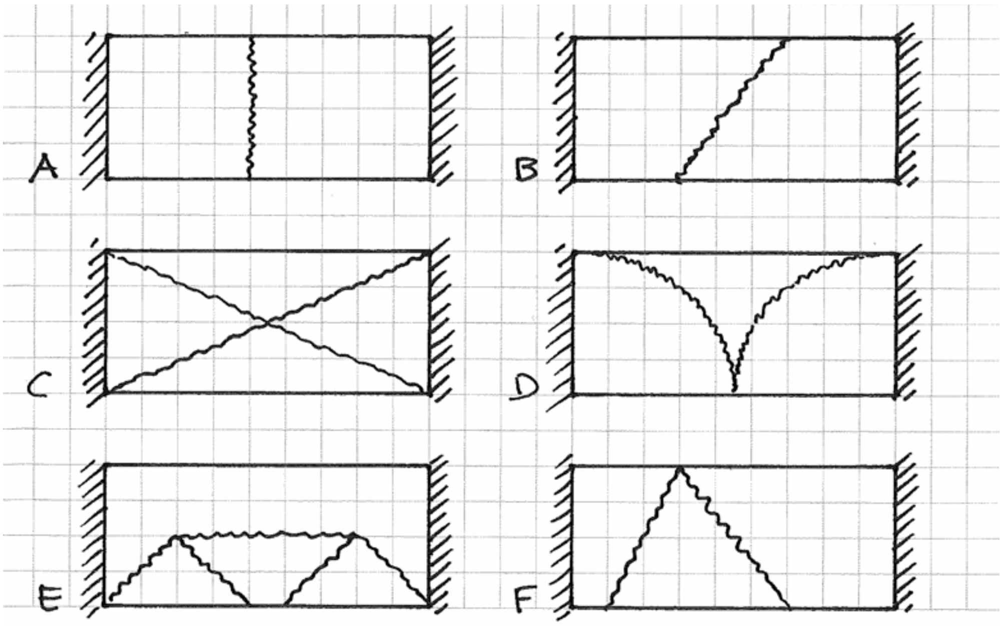

Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 4. Determine an upper bound for q expressed in m_p and a (1.5 point).

c Determine the largest lower-bound for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a The upper bound theorem of plasticity theory … Choose A, B, C, or D (0.5 point).

A … is generally believed to be true but it has not been mathematically proved as yet.
B … has been mathematically proved by W. Prager in 1950.
C … has been confirmed by many experiments but is unlikely to be proved mathematically because clear exceptions have been found, such as simply supported reinforced concrete beams.
D … has been formulated and proved by Vitruvius (~ 85 – 20 BC).

b Which yield condition depends on two experimentally determined numbers? Choose A, B, C, or D (0.5 point).

A Von Mises yield condition,
B Tresca yield condition,
C Mohr-Coulomb yield condition,
D Saint Venant yield condition.

c Temperature stresses do not influence the strength of ductile structures. Why is this? Choose A, B, C or D (0.5 point).

A Because temperature stresses are small and can be neglected.
B Because the plastic hinges make the structure statically determinate and in a statically determinate structure, temperature stresses do not occur.
C Because temperature loading is compensated by creep. In the short term it might have some influence but in the long term it has not.
D Temperature stress influences the deformations and if the applied dilatations are not sufficient it can lead to premature failure.

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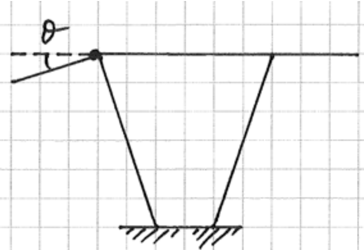
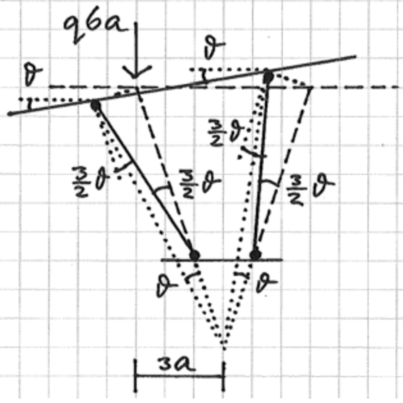
Answer to Problem 1a



$$E = 2 M_p \vartheta$$

$$A = q\, 3a\, \vartheta\, \tfrac{3}{2} a$$

$$E = A \Rightarrow \quad q = \frac{4}{9}\frac{M_p}{a^2}$$



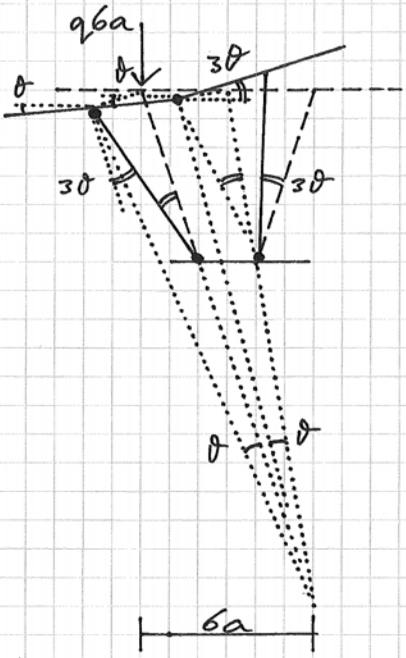
$$E = M_p \tfrac{3}{2}\vartheta + M_p\left(\tfrac{3}{2}\vartheta - \vartheta\right) +$$

$$+ M_p\left(\tfrac{3}{2}\vartheta - \vartheta\right) + M_p \tfrac{3}{2}\vartheta$$

$$A = q\, 6a\, \vartheta\, 3a$$

$$E = A \Rightarrow \boxed{q = \frac{2}{9}\frac{M_p}{a^2}}$$

decisive



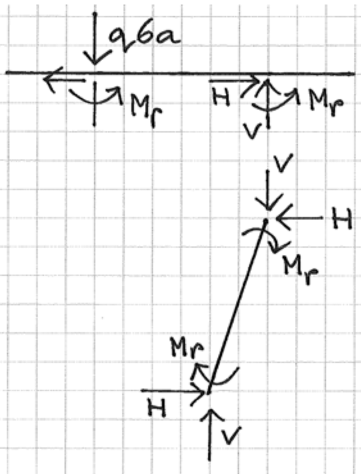
$$E = M_p 3\vartheta + M_p(3\vartheta - \vartheta) +$$

$$2M_p(3\vartheta - \vartheta) + M_p 3\vartheta$$

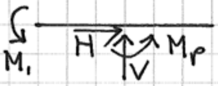
$$A = q\, 6a\, \vartheta\, 6a$$

$$E = A \Rightarrow \quad q = \frac{1}{3}\frac{M_p}{a^2}$$

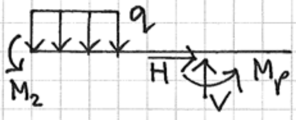
Answer to Problem 1b



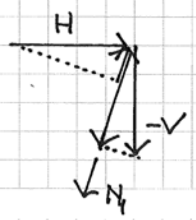
$$\left.\begin{aligned} 2M_p + V6a &= 0 \\ 2M_p + V2a - H6a &= 0 \end{aligned}\right\} \quad \begin{aligned} H &= \frac{2}{9}\frac{M_p}{a} \\ V &= -\frac{1}{3}\frac{M_p}{a} \end{aligned}$$



$$M_1 = -V3a - M_p = 0$$



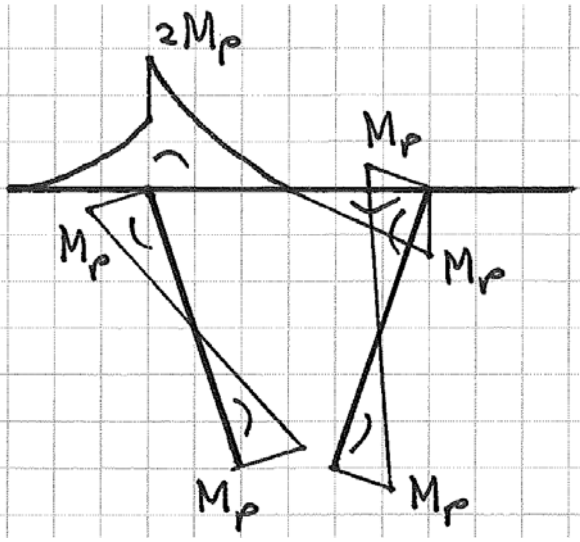
$$M_2 = q3a\frac{3a}{2} - V6a - M_p = 2M_p$$

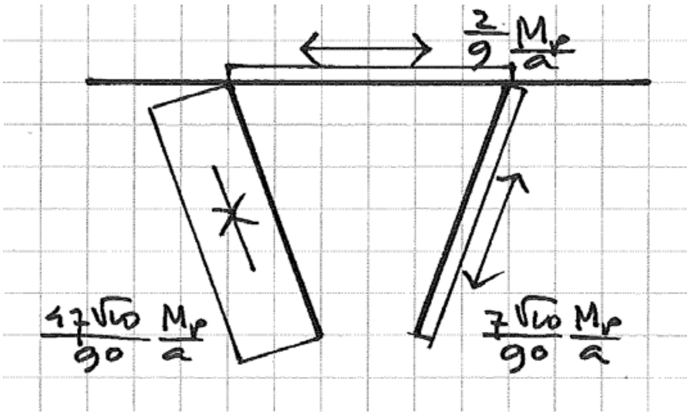


$$N_1 = -V\frac{3}{\sqrt{10}} - H\frac{1}{\sqrt{10}} = \frac{7\sqrt{10}}{90}\frac{M_p}{a}$$

$q6a - V = \frac{5}{3}\frac{M_p}{a}$

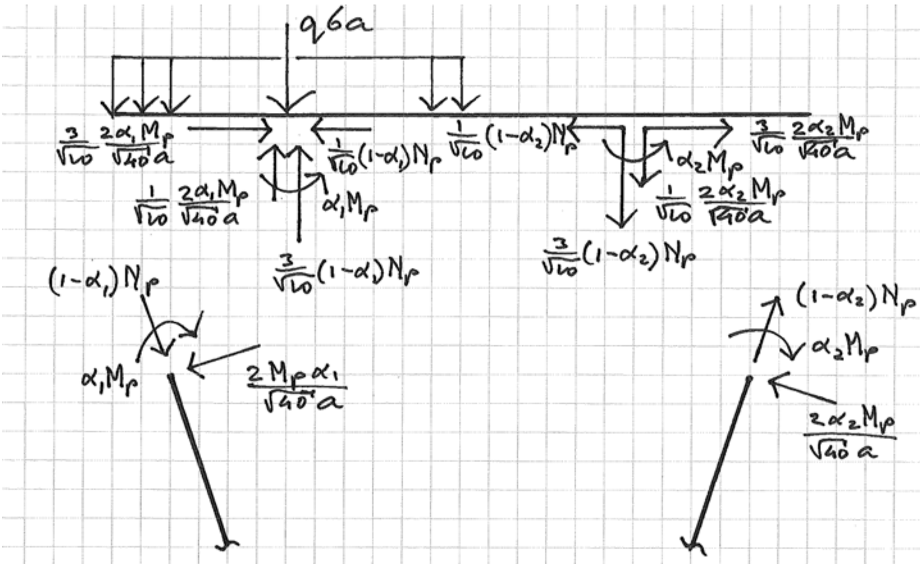
$$N_2 = H\frac{1}{\sqrt{10}} + \frac{5}{3}\frac{M_p}{a}\frac{3}{\sqrt{10}} = \frac{47\sqrt{10}}{90}\frac{M_p}{a}$$





Answer to Problem 1c

lower-bound



verticaal evenwicht ↓

$$q6a - \frac{1}{\sqrt{10}}\frac{2\alpha_1 M_p}{\sqrt{40}\,a} - \frac{3}{\sqrt{10}}(1-\alpha_1)N_p + \frac{3}{\sqrt{10}}(1-\alpha_2)N_p + \frac{1}{\sqrt{10}}\frac{2\alpha_2 M_p}{\sqrt{40}\,a} = 0$$

horizontaal evenwicht →

$$\frac{3}{\sqrt{10}}\frac{2\alpha_1 M_p}{\sqrt{40}\,a} - \frac{1}{\sqrt{10}}(1-\alpha_1)N_p - \frac{1}{\sqrt{10}}(1-\alpha_2)N_p + \frac{3}{\sqrt{10}}\frac{2\alpha_2 M_p}{\sqrt{40}\,a} = 0$$

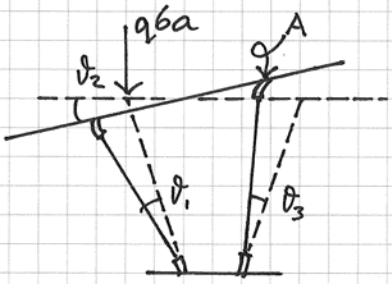
momenten evenwicht ↶

$$\alpha_1 M_p + \alpha_2 M_p - 6a\left[\frac{3}{\sqrt{10}}(1-\alpha_2)N_p + \frac{1}{\sqrt{10}}\frac{2\alpha_2 M_p}{\sqrt{40}\,a}\right] = 0$$

The solution to the equations is

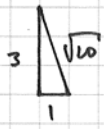
$$\alpha_1 = \frac{7870}{8339}, \quad \alpha_2 = \frac{8270}{8339}, \quad q = \frac{20}{93}\frac{M_p}{a^2}$$

upper-bound



$u = \frac{a}{\beta}\vartheta$

$\beta = 9\sqrt{10}$



horizontal displacement of A ←

$$-\vartheta_1 \frac{a}{\beta}\frac{1}{\sqrt{10}} + \vartheta_1 6a - (\vartheta_1-\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{10}} = -\vartheta_3\frac{a}{\beta}\frac{1}{\sqrt{10}} + \vartheta_3 6a - (\vartheta_3-\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{10}}$$

vertical displacement of A ↑

$$-\vartheta_1\frac{a}{\beta}\frac{3}{\sqrt{10}} - \vartheta_1 2a - (\vartheta_1-\vartheta_2)\frac{a}{\beta}\frac{3}{\sqrt{10}} + \vartheta_2 6a = \vartheta_3\frac{a}{\beta}\frac{3}{\sqrt{10}} + \vartheta_3 2a + (\vartheta_3-\vartheta_2)\frac{a}{\beta}\frac{3}{\sqrt{10}}$$

$$E = M_p\vartheta_1 + M_p(\vartheta_1-\vartheta_2) + M_p(\vartheta_3-\vartheta_2) + M_p\vartheta_3$$

$$A = q6a\left[\vartheta_1\frac{a}{\beta}\frac{3}{\sqrt{10}} + \vartheta_1 2a + (\vartheta_1-\vartheta_2)\frac{a}{\beta}\frac{3}{\sqrt{10}}\right]$$

$$E = A$$

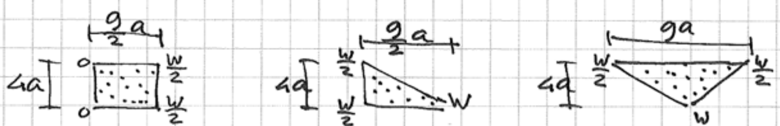
The solution to the equatons is

$$\vartheta_1 = \frac{91}{62}\vartheta_2\ ,\quad \vartheta_3 = \frac{91}{62}\vartheta_2\ ,\quad q = \frac{20}{93}\frac{M_p}{a^2} \approx 0.215\frac{M_p}{a^2}$$

Answer to Problem 2a

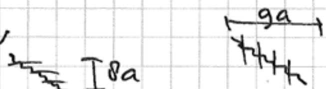
A, C, E, F

Answer to Problem 2b



$$A = 2\left\{4a\frac{9}{2}aq\frac{1}{4}w + \frac{1}{2}4a\frac{9}{2}aq\frac{2}{3}w\right\} + \frac{1}{2}4a9aq\frac{2}{3}w$$

$$= 33\,a^2qw$$

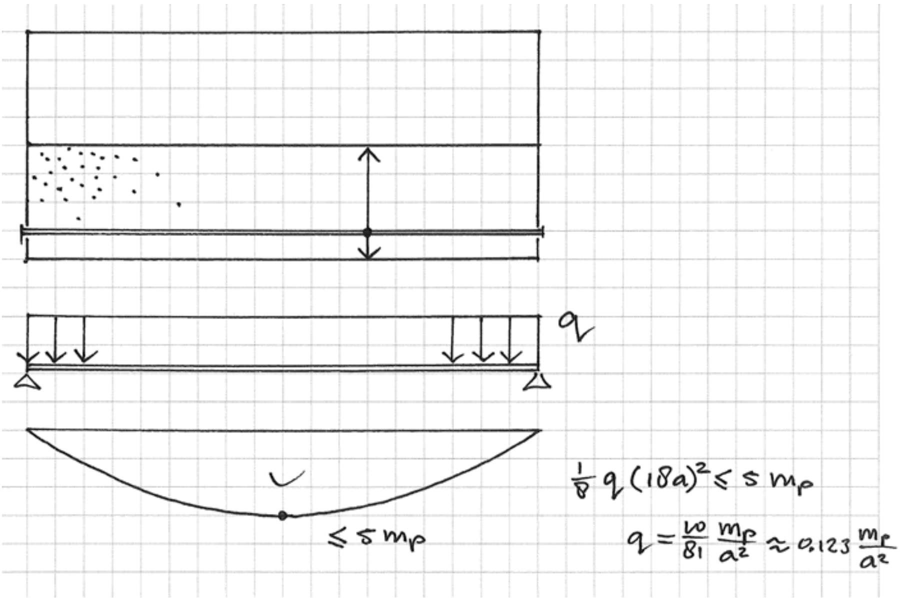


$$E = 2\left\{5m_p 8a\frac{w}{9a} + m_p 9a\frac{w}{8a}\right\}$$

$$= \frac{401}{36}m_p w$$

$$A = E \quad\Rightarrow\quad q = \frac{401}{1188}\frac{m_p}{a^2} \approx 0.338\frac{m_p}{a^2}$$

Answer to Problem 2c



Answer to Problem 3a
B

Answer to Problem 3b
C

Answer to Problem 3c
B