Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 20 January 2011, 14:00 – 17:00 hours

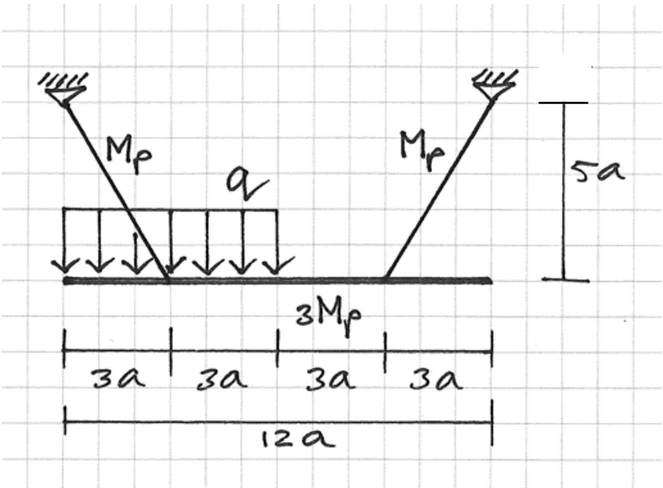

Figure 1. Frame structure

Problem 1

A frame consists of two hangers and a beam (Fig. 1). The beam is three times as strong as the hangers. The joints between the members are fixed connections. The structure is loaded by a vertical load q that is evenly distributed over the left half of the beam. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

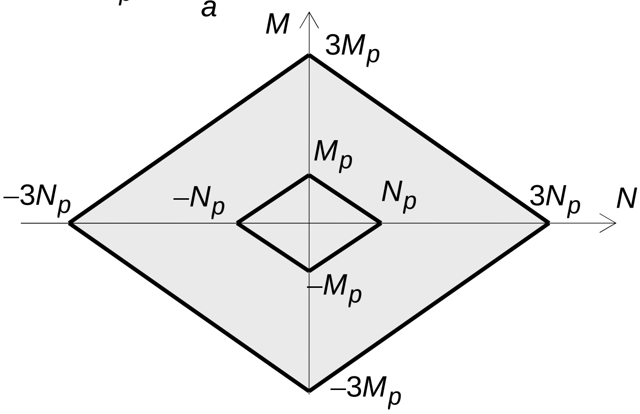

Figure 2. Yield contours

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = \dfrac{60}{\sqrt{34}}$. Choose one of the following problems (You need not do both).
– Use Fig. 3 to determine the largest <u>lower-bound</u> for *q*.
– Determine the smallest <u>upper-bound</u> for *q*.
You only need to write down the equations and not solve the equations (1.5 points).

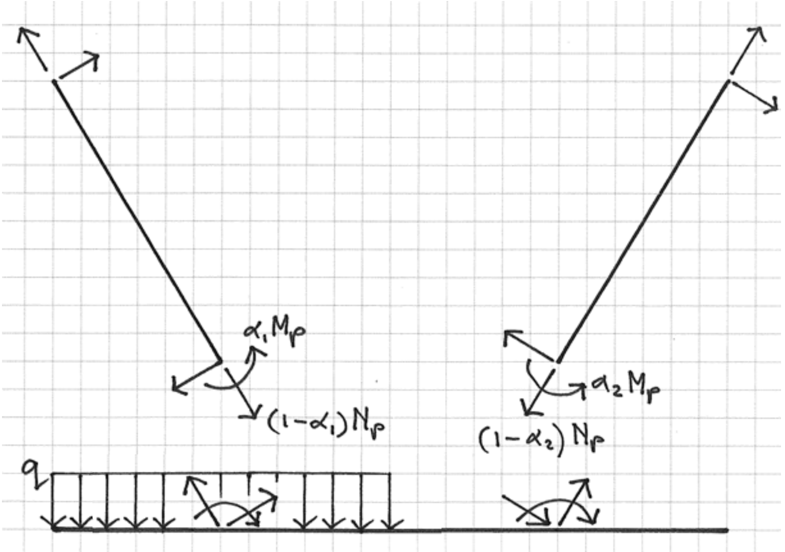

Figure 3. Equilibrium system for including M-N interaction

Problem 2

A reinforced concrete plate has a simply supported edge and a fixed edge (Fig. 4). In the middle it has a large opening. It carries an evenly distributed load *q*. The plate is homogeneous and orthotropic.

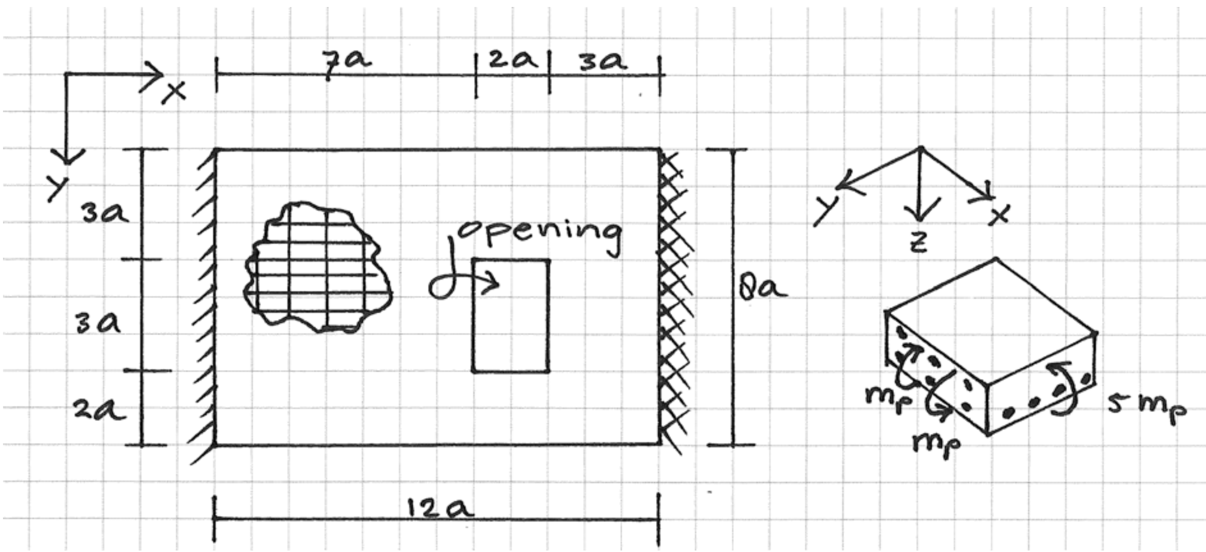

Figure 4. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

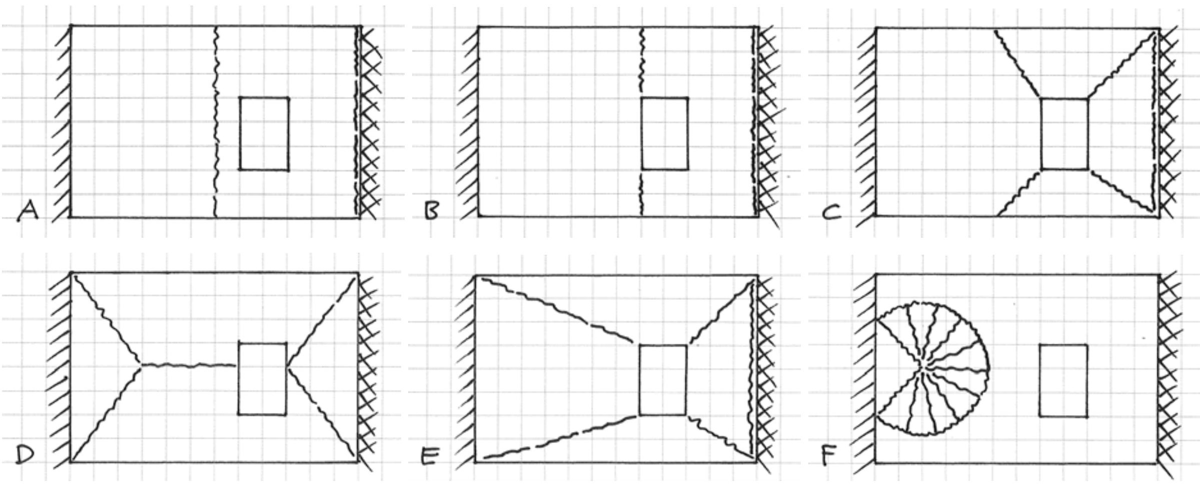

Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 6. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

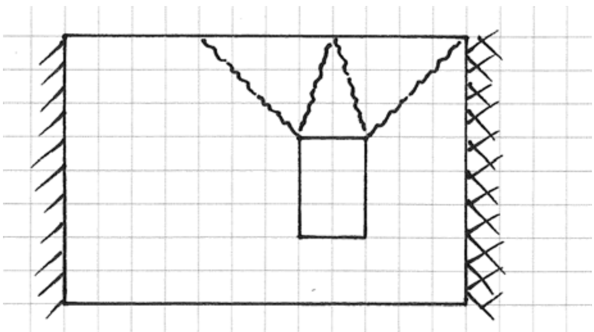

Figure 6. Yield line pattern of problem 2b

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Which is the lower bound theorem of plasticity theory? Choose A, B, C, or D (0.5 point).

A Any equilibrium system that fulfils the yield conditions gives a safe approximation to the strength of a structure,

B Any equilibrium system that fulfils the yield conditions gives a good approximation to the strength of a structure,

C Any kinematically admissible equilibrium system gives an unsafe approximation to the strength of a structure,

D Any equilibrium system that fulfils the yield conditions and the kinematic boundary conditions gives an accurate and safe approximation to the strength of a structure.

b Plastic structural components have a convex yield contour. Why is this?
Choose A, B, C, or D (0.5 point).

A Because for a plastic yield contour just a few load combinations are needed to make sure that a structure is safe.
B Because otherwise the components might fail in a brittle way.
C Because this has been demonstrated in many experiments.
D Because a mechanism gives a straight line in the graph of the yield contour and many straight lines enclose an area that is convex.

c The Von Mises yield condition is not suitable for reinforced concrete? Why is this?
Choose A, B, C or D (0.5 point).

A Because reinforced concrete is not a plastic material.
B Because reinforced concrete does not fail due to the largest shear stress.
C Because this has been demonstrated in many experiments.
D Because reinforced concrete consists of two materials which need to be modelled individually.

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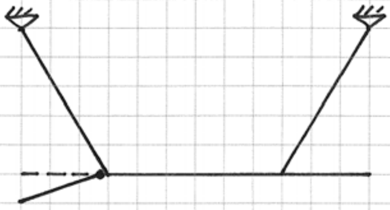
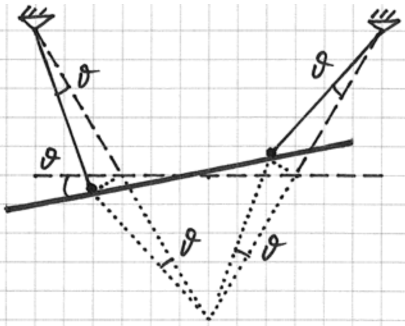
Answer to Problem 1a



$$E = 3 M_p \vartheta$$

$$A = q\, 3a\, \tfrac{3}{2} a\, \vartheta$$

$$E = A \;\Rightarrow\; q = \frac{2}{3}\frac{M_p}{a^2}$$

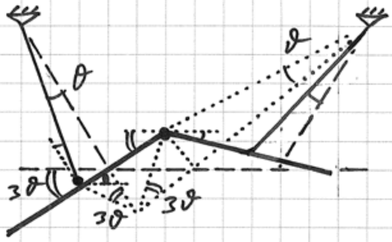


$$E = M_p(\vartheta+\vartheta) + M_p(\vartheta+\vartheta)$$

$$A = q\, 6a\, \vartheta\, 3a$$

$$E = A \;\Rightarrow\; \boxed{q = \frac{2}{9}\frac{M_p}{a^2}}$$

decisive

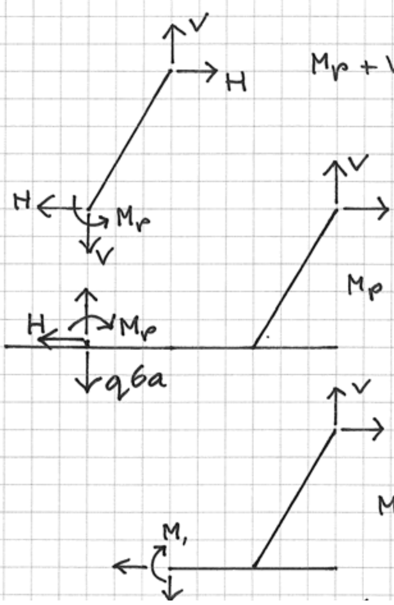


$$E = M_p(\vartheta+3\vartheta) + 3M_p(3\vartheta+\vartheta)$$

$$A = q\, 6a\, \vartheta\, 3a$$

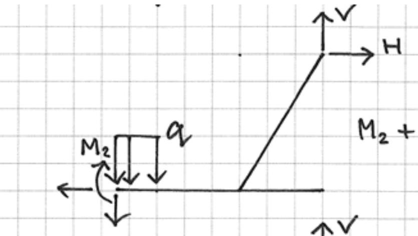
$$E = A \;\Rightarrow\; q = \frac{8}{9}\frac{M_p}{a^2}$$

Answer to Problem 1b

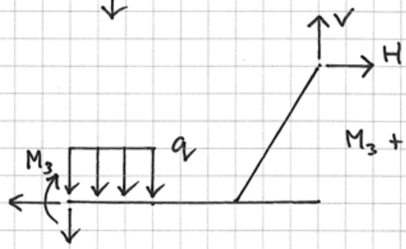


$$\left.\begin{aligned} M_p + V3a - H5a &= 0 \\ M_p + H5a - V9a &= 0 \end{aligned}\right\} \quad H = \frac{2}{5}\frac{M_p}{a}, \quad V = \frac{1}{3}\frac{M_p}{a}$$

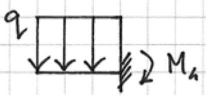
$$M_1 + H5a - V6a = 0 \;\Rightarrow\; M_1 = 0$$



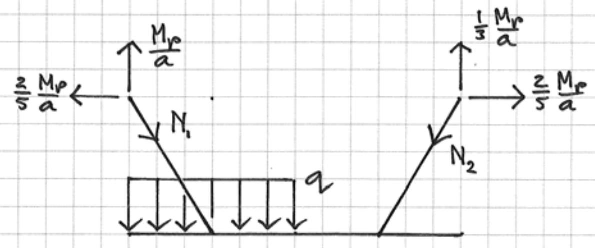
$$M_2 + H5a - V\frac{15}{2}a + q\frac{3}{2}a\frac{3}{4}a = 0$$
$$\Rightarrow M_2 = \frac{1}{4}M_p$$



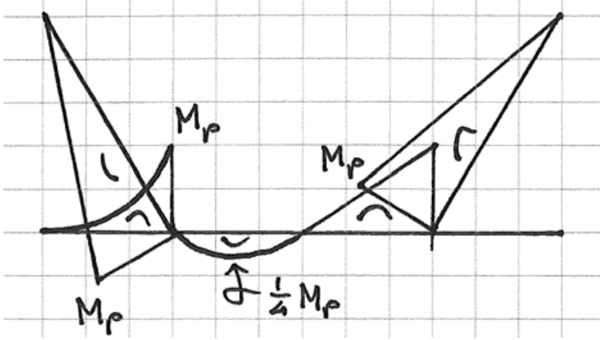
$$M_3 + H5a - V9a + q\,3a\frac{3}{2}a = 0$$
$$\Rightarrow M_3 = 0$$

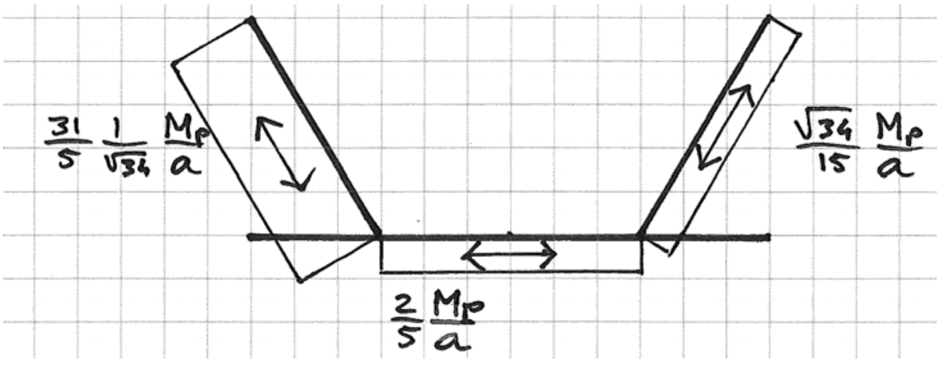


$$M_4 = q\,3a\frac{3}{2}a \Rightarrow M_4 = M_p$$



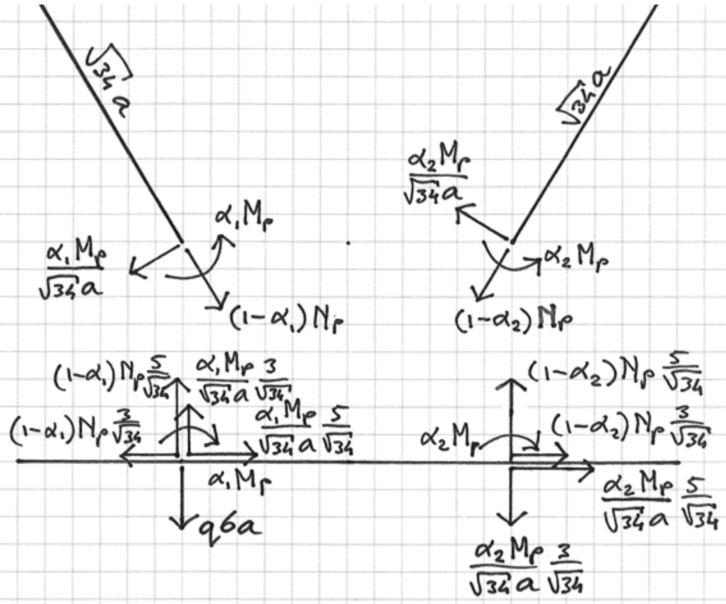
$$N_1 = \frac{M_p}{a}\frac{5}{\sqrt{34}} + \frac{2}{5}\frac{M_p}{a}\frac{3}{\sqrt{34}} = \frac{31}{5}\frac{1}{\sqrt{34}}\frac{M_p}{a}$$
$$N_2 = \frac{1}{3}\frac{M_p}{a}\frac{5}{\sqrt{34}} + \frac{2}{5}\frac{M_p}{a}\frac{3}{\sqrt{34}} = \frac{\sqrt{34}}{15}\frac{M_p}{a}$$





Answer to Problem 1c

lower-bound



vert. $q6a - (1-\alpha_1)N_p\frac{5}{\sqrt{34}} - \frac{\alpha_1 M_p}{\sqrt{34}a}\frac{3}{\sqrt{34}} + \frac{\alpha_2 M_p}{\sqrt{34}a}\frac{3}{\sqrt{34}} - (1-\alpha_2)N_p\frac{5}{\sqrt{34}} = 0$

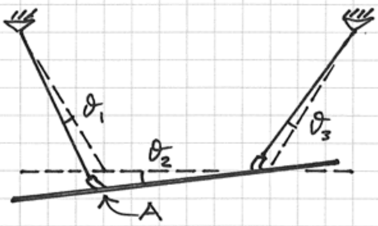
hor. $-(1-\alpha_1)N_p\frac{3}{\sqrt{34}} + \frac{\alpha_1 M_p}{\sqrt{34}a}\frac{5}{\sqrt{34}} + (1-\alpha_2)N_p\frac{3}{\sqrt{34}} + \frac{\alpha_2 M_p}{\sqrt{34}a}\frac{5}{\sqrt{34}} = 0$

mom. $\alpha_1 M_p + \alpha_2 M_p + 6a\left[\frac{\alpha_2 M_p}{\sqrt{34}a}\frac{3}{\sqrt{34}} - (1-\alpha_2)N_p\frac{5}{\sqrt{34}}\right] = 0$

The solution to the equations is

$\alpha_1 = \frac{3500}{3873} \qquad \alpha_2 = \frac{3700}{3873} \qquad q = \frac{800}{3873}\frac{M_p}{a^2} \approx 0.207\frac{M_p}{a^2}$

upper-bound



$u = \vartheta\frac{a}{\beta}$

horizontal displacement of A ←

$\vartheta_1 5a - (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{3}{\sqrt{34}} = \vartheta_3 5a + (\vartheta_2+\vartheta_3)\frac{a}{\beta}\frac{3}{\sqrt{34}}$

vertical displacement of A ↓

$\vartheta_1 3a + (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{5}{\sqrt{34}} = -\vartheta_3 3a + (\vartheta_2+\vartheta_3)\frac{a}{\beta}\frac{5}{\sqrt{34}} + \vartheta_2 6a$

$E = M_p(\vartheta_1+\vartheta_2) + M_p(\vartheta_2+\vartheta_3)$

$A = q6a\left[\vartheta_1 3a + (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{5}{\sqrt{34}}\right]$

$E = A$

The solution to the equations is

$\vartheta_1 = \frac{3671}{3601}\vartheta_2\,, \quad \vartheta_3 = \frac{3527}{3601}\vartheta_2\,, \quad q = \frac{800}{3873}\frac{M_p}{a^2}$

Answer to Problem 2a
A, B, C, E, F

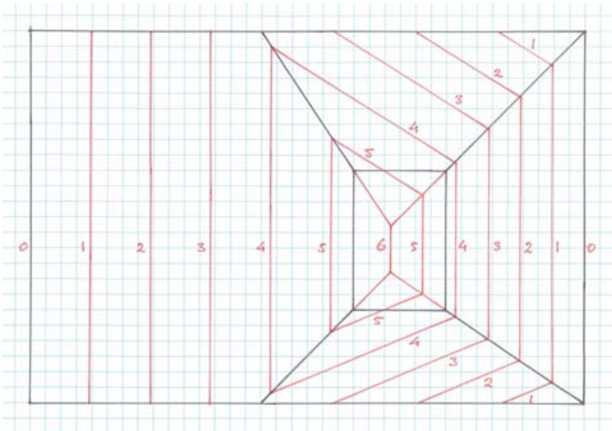


Figure 7. Deformation of pattern C

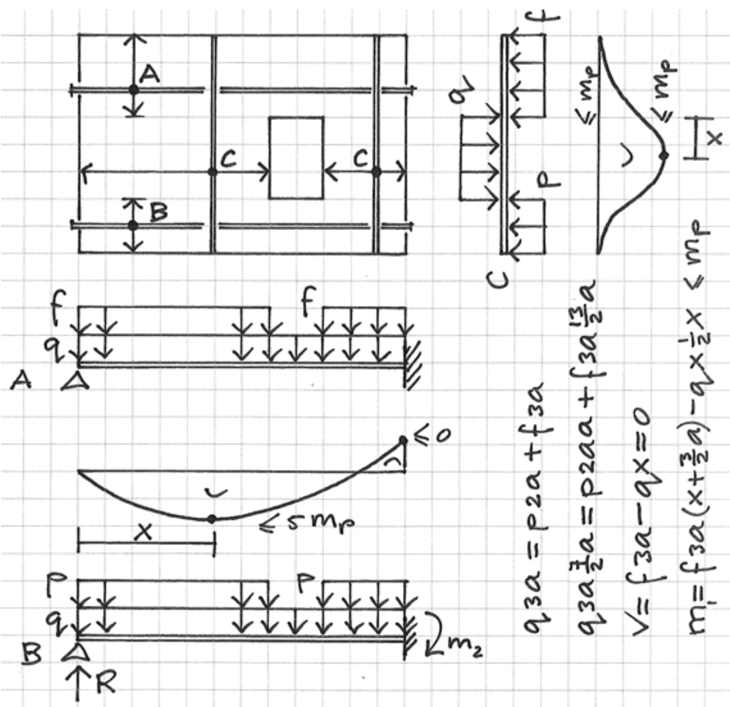
Answer to Problem 2b

$$E = 2\left[m_p 3a \frac{w}{4a}\right] + 2\left[m_p a \left(\frac{w}{3a} - \frac{w}{4a}\right) + 5 m_p 3a \frac{w}{4a}\right]$$

$$A = 2\left[\frac{1}{2} 4a\, 3a\, q \frac{w}{3}\right] + \frac{1}{2} 2a\, 3a\, q \frac{w}{3}$$

$$E = A \Rightarrow q = \frac{11}{6} \frac{m_p}{a^2} \approx 1{,}83 \frac{m_p}{a^2}$$

Answer to Problem 2c



$$m_2 = -R\,12a + q\,12a\,6a + p\,7a\,\frac{17}{2}a + p\,3a\,\frac{3}{2}a \leqslant 0$$

$$V = R - (p+q)x = 0$$

$$m_3 = Rx - (q+p)x\,\frac{1}{2}x \leqslant 5\,m_p$$

The solution to the equations is $q = \frac{550}{3249}\frac{m_p}{a^2} \approx 0.17\frac{m_p}{a^2}$

Answer to Problem 3a
A

Answer to Problem 3b
D

Answer to Problem 3c
C