Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 12 April 2012, 14:00 – 17:00 hours

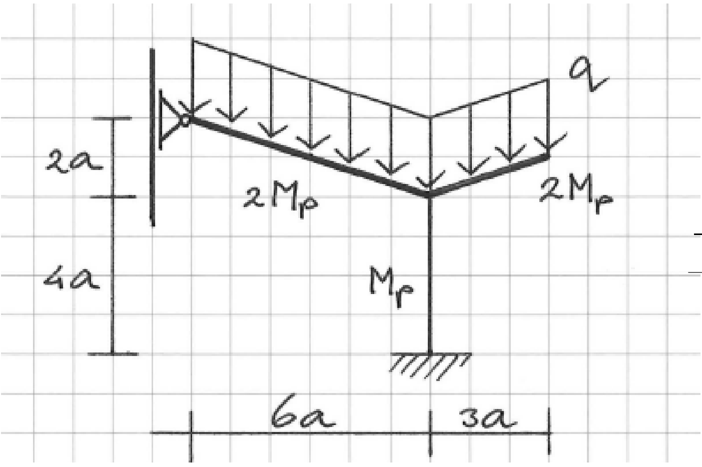

Figure 1. Frame structure

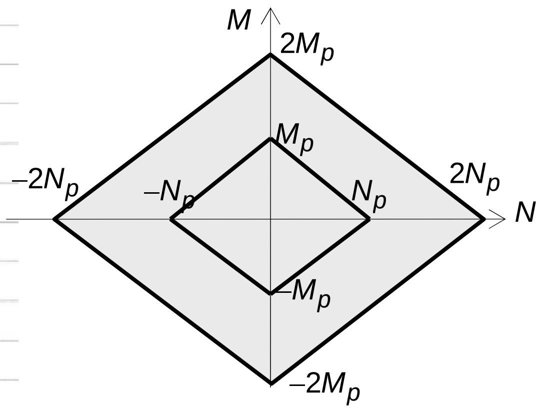

Figure 2. Yield contour

Problem 1

A frame consists of a column with a strength M_p and two beams with strengths $2M_p$ (Fig. 1). The joints are fixed connections. The left beam is supported by a roller. The structure is loaded by a vertical load q (self weight). The relation of Figure 2 exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 30. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for p.
– Determine the smallest upper-bound for p.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has 8 simply supported edges (Fig. 3). It carries an evenly distributed load p. The plate is homogeneous and orthotropic. The bottom reinforcement gives a yield moment of $2m_p$ in both directions. The top reinforcement gives a yield moment of m_p in both directions.

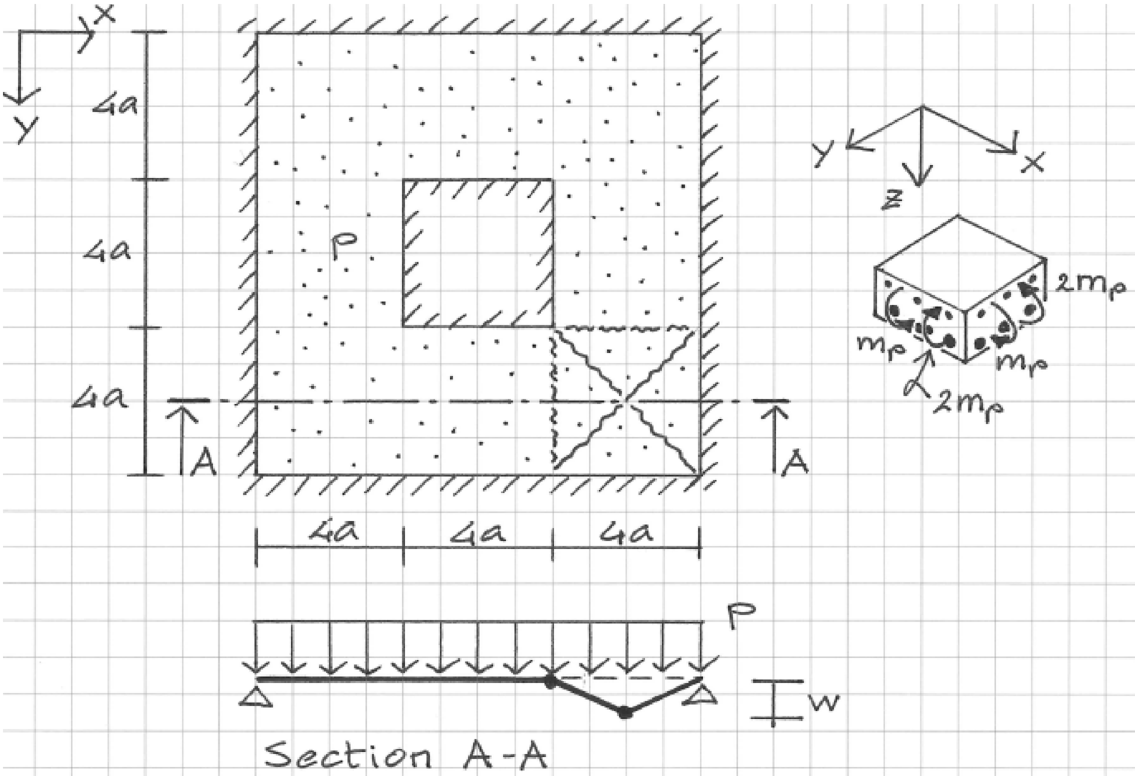

Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

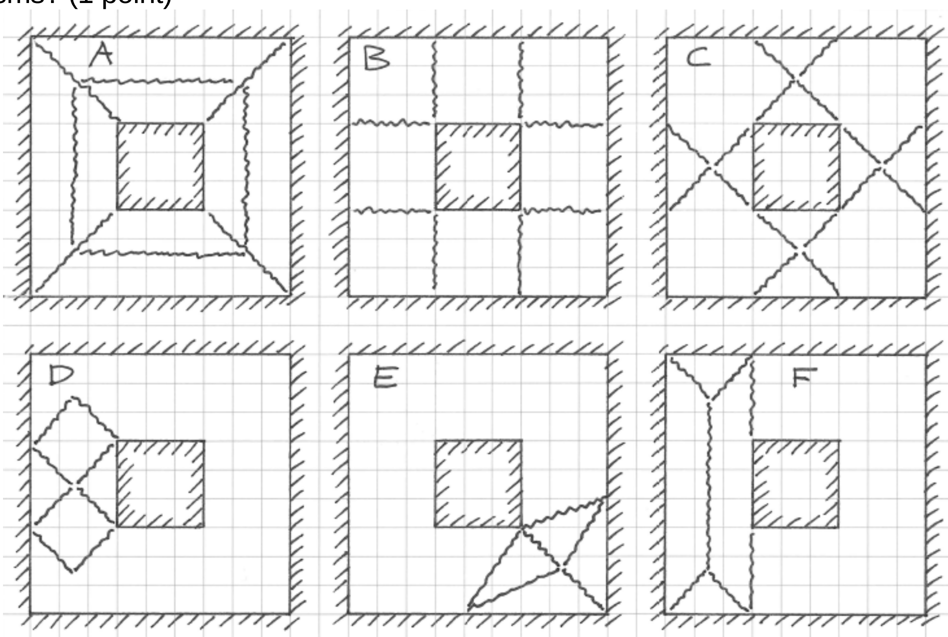

Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for p expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for p using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Which limit states of a structure can be checked using plastic analysis?
Choose from A to F (0.5 point).

A Fatigue
B Strength
C Deflection
D Vibrations
E Stability
F Crack widths in reinforced concrete

b Can plasticity theory be used to analyse glass beams? Explain your answer (0.5 point).

c The virtual work equation replaces the ... Choose A, B, C or D for a correct ending.
(0.5 point).

A … kinematic equations.
B … equilibrium equations.
C … compatibility equation.
D … constitutive equations.

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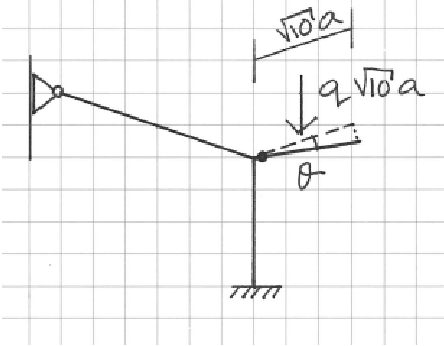
Answer to Problem 1a

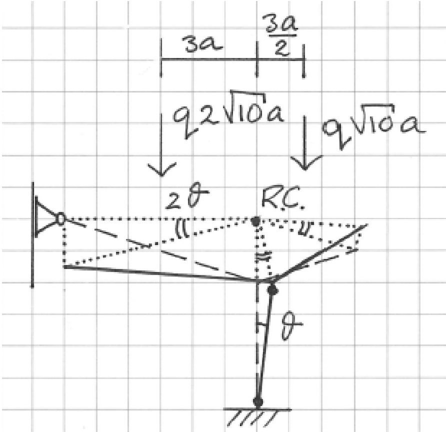


$$A = q\sqrt{10}a\,\frac{3a}{2}\,\theta$$

$$E = 2M_p\theta$$

$$E = A \Rightarrow q = \frac{2}{15}\sqrt{10}\,\frac{M_p}{a^2}$$

$$= 0.42\,\frac{M_p}{a^2}$$

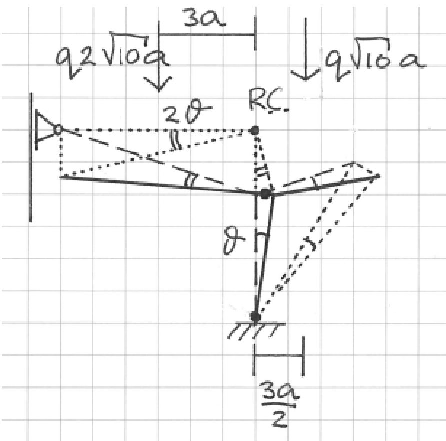


$$A = q2\sqrt{10}a\,2\theta\,3a - q\sqrt{10}a\,2\theta\,\frac{3a}{2}$$

$$E = M_p\theta + M_p(\theta + 2\theta)$$

$$E = A \Rightarrow \boxed{q = \frac{2}{45}\sqrt{10}\,\frac{M_p}{a^2}} \quad \text{decisive}$$

0.14



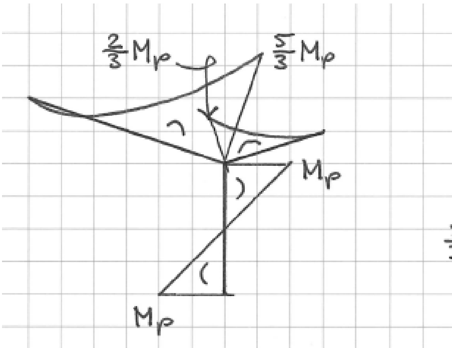
$$A = q2\sqrt{10}a\,2\theta\,3a + q\sqrt{10}a\,\theta\,\frac{3a}{2}$$

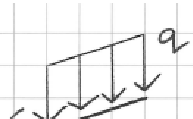
$$E = M_p\theta + 2M_p(\theta + 2\theta)$$

$$E = A \Rightarrow q = \frac{7}{135}\sqrt{10}\,\frac{M_p}{a^2}$$

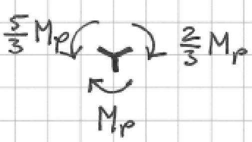
0.16

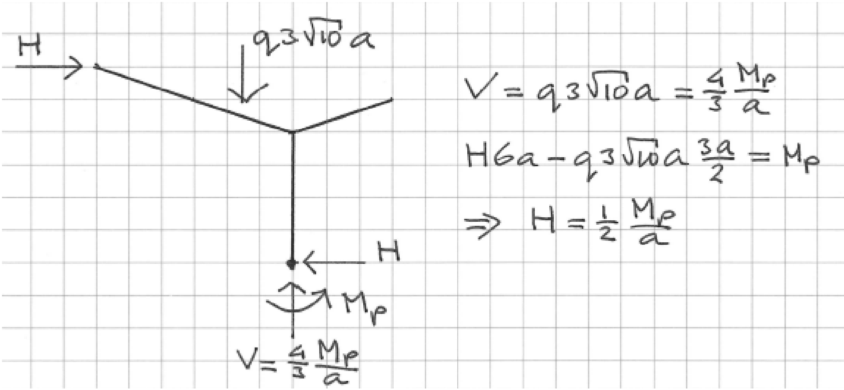
Answer to Problem 1b

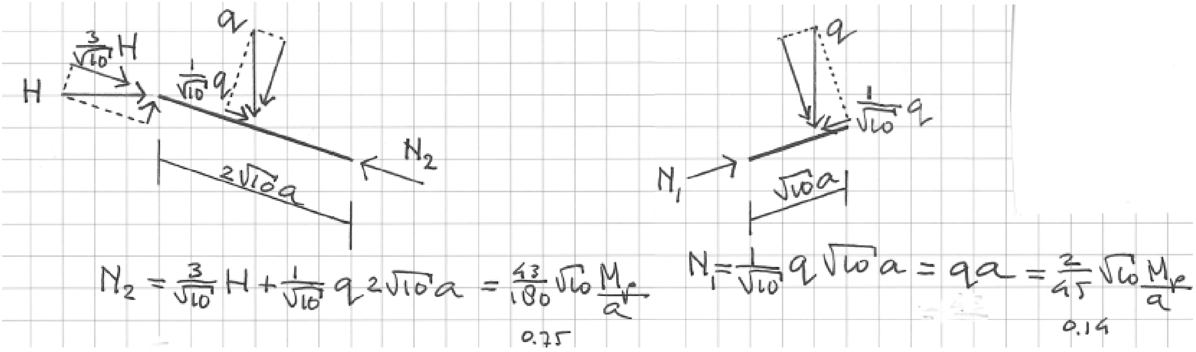


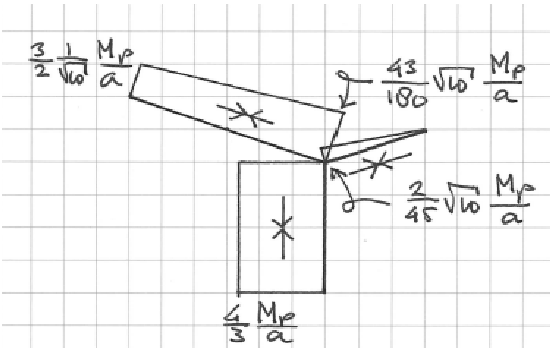


$$M_1 = q\sqrt{10}a\,\frac{3a}{2} = \frac{2}{3}M_p$$



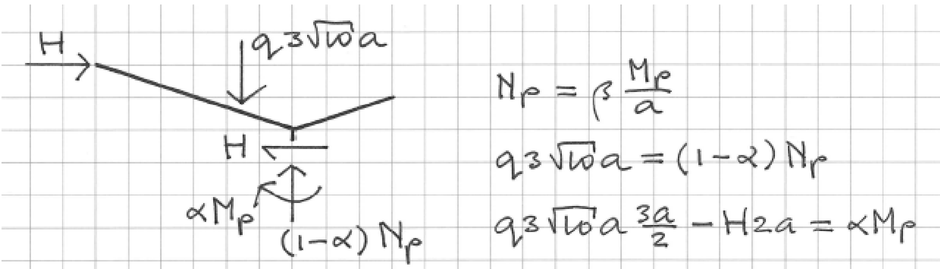


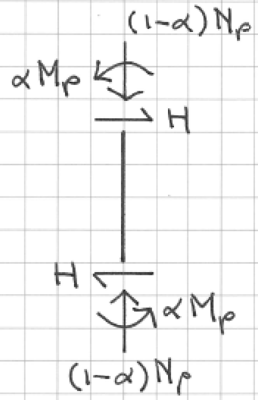




Answer to Problem 1c

lower-bound



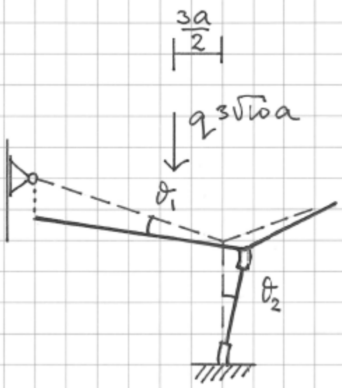


$$H\,4a = 2\alpha M_p$$

The solution to the equations is

$$q = \frac{2}{47}\sqrt{10}\,\frac{M_p}{a^2} \qquad \alpha = \frac{45}{47} \qquad H = \frac{45}{49}\,\frac{M_p}{a}$$

upper-bound



$\beta = 30$

$u = \theta \frac{a}{\beta}$

hor. disp. of the roller is zero

$$\theta_2 4a - \theta_1 2a = 0$$

$$E = M_p\,\theta_2 + M_p(\theta_1 + \theta_2)$$

$$A = q\,3\sqrt{10}\,a\left[\theta_2\frac{a}{\beta} + (\theta_1+\theta_2)\frac{a}{\beta} + \theta_1\frac{3a}{2}\right]$$

$$E = A \;\Rightarrow\; q = \frac{2}{47}\sqrt{10}\,\frac{M_p}{a^2}$$

0.13

Answer to Problem 2a

A, E, F

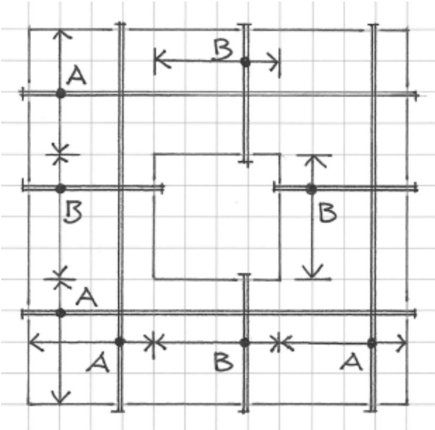
Answer to Problem 2b

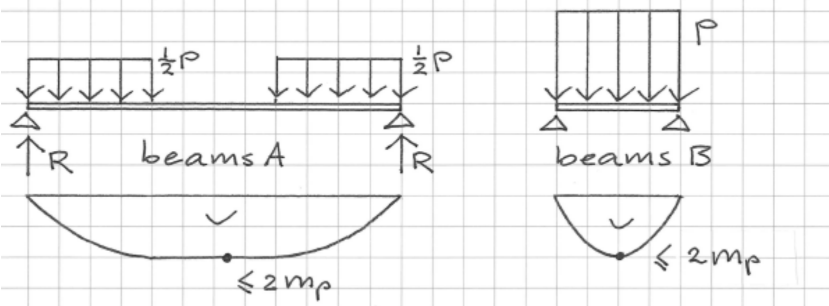
$$E = 2\left[m_p\frac{w}{2a}4a\right] + 4\left[2m_p\frac{w}{2a}2a + 2m_p\frac{w}{2a}2a\right]$$

$$A = 4\left[p\,\frac{1}{2}4a\,2a\,\frac{w}{3}\right]$$

$$E = A \;\Rightarrow\; p = \frac{15}{4}\,\frac{m_p}{a^2}$$

Answer to Problem 2c





$2\,\frac{1}{2}p\,4a = 2R$

$\frac{1}{8}p\,(4a)^2 \leqslant 2m_p$

$m = R\,6a - \frac{1}{2}p\,4a\,4a \leqslant 2m_p$

The solution to the equations is

$p = \frac{1}{2}\frac{m_p}{a^2}$

Answer to Problem 3a
B

Answer to Problem 3b
No, glass is a brittle material.
or
Yes, reinforced glass can be sufficiently ductile.

Answer to Problem 3c
B