Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 26 January 2012, 14:00 – 17:00 hours

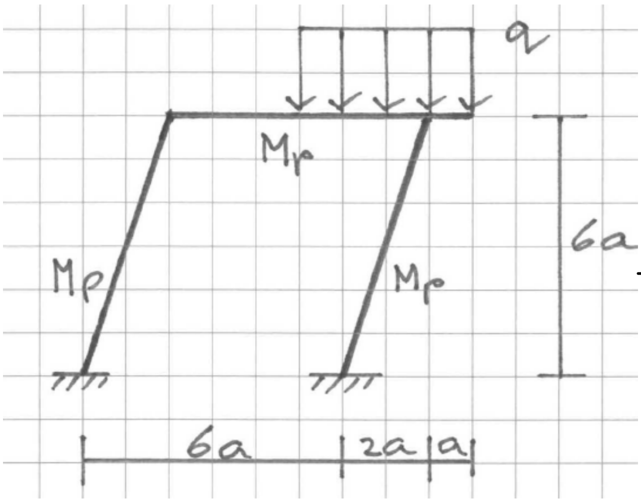


Figure 1. Frame structure

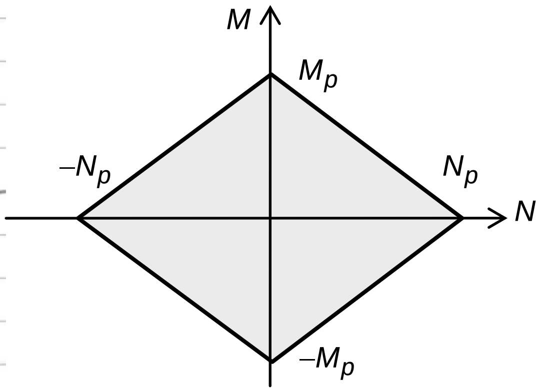


Figure 2. Yield contour

Problem 1

A frame consists of two columns and a beam (Fig. 1). The joints are fixed connections. The structure is loaded by a vertical load *q*. The relation of Figure 2 exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load *q* for all possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 10. Choose one of the following problems (You need not do both).
– Use Figure 3 to determine the largest lower-bound for *q*.
– Determine the smallest upper-bound for *q*.
You only need to write down the equations and not solve the equations (1.5 points).

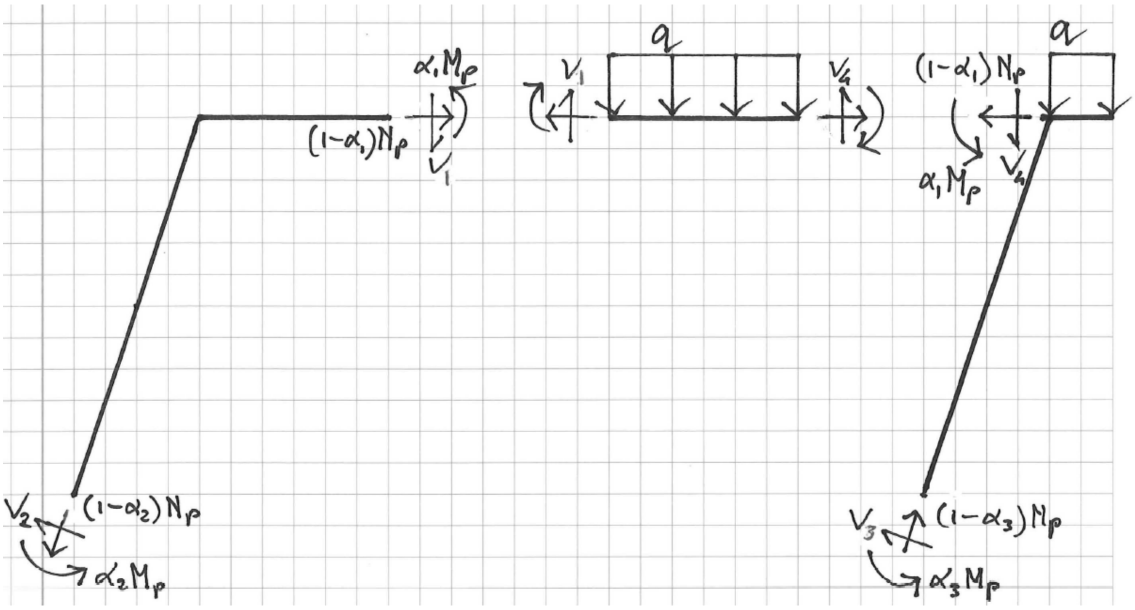


Figure 3. Equilibrium system for including M-N interaction

Problem 2

A reinforced concrete plate has two simply supported edges (Fig. 4). It carries an evenly distributed load *q* over half of the plate (shaded area). The plate is homogeneous and orthotropic.

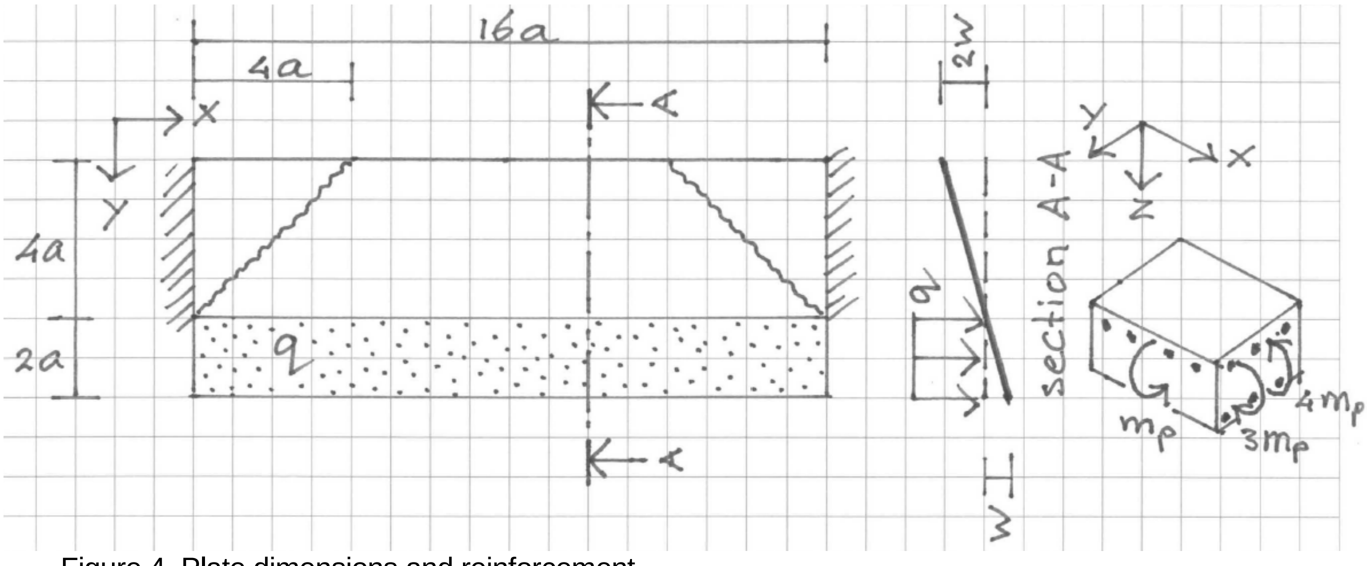


Figure 4. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

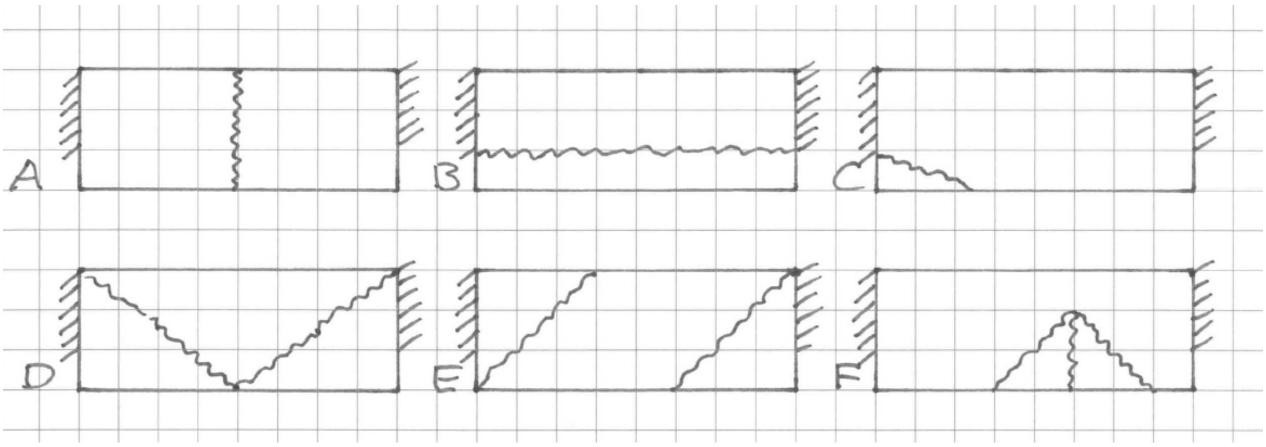


Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 4. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a What is the reason for using the rhombic interaction diagram of Figure 2?
Choose A, B, C, or D (0.38 point).

A This diagram is not so realistic for many sections but always on the safe side.
B This diagram is exact for rectangular cross-sections.
C This diagram is a very good approximation for **I**-sections which are often applied.
D This diagram is mathematically consistent to the virtual work equation.

b Which of the following words refers to a different concept than the others?
Choose A, B, C, or D (0.38 point).

A Interaction diagram
B Deformation capacity
C Yield contour
D Limit state function

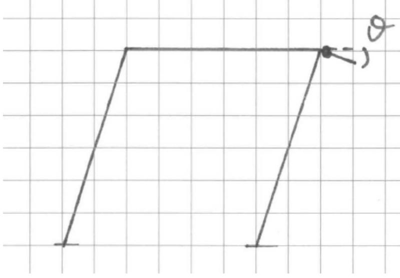
c The deflection of the centre of gravity of a rigid triangular plate is ...
Choose A, B, C or D (0.38 point).

A … one third of the largest deflection.
B … half of the average of the two largest corner deflections.
C … the quotient of the static moment and the surface area.
D … the average of the corner deflections.

d The strength of reinforced concrete plates loaded in bending can be much higher than found in a plastic analysis such as problem 2b. What causes this? Choose A, B, C, or D (0.38 point).

A Membrane action and arch action.
B It is an upper bound analysis and the real strength can be larger.
C Safe assumptions are made on the material strengths.
D The virtual work equation provides only an approximation of equilibrium.

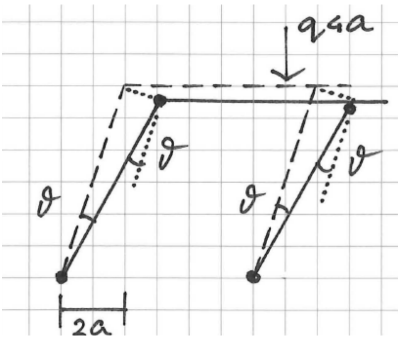
Answer to Problem 1a



$$E = M_p \vartheta$$

$$A = q a \tfrac{1}{2} a \vartheta$$

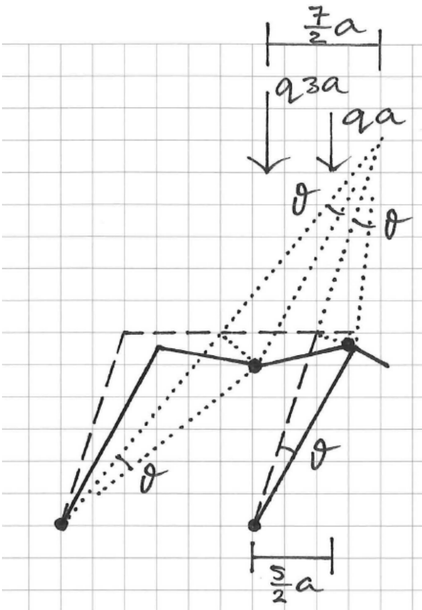
$$E = A \Rightarrow q = 2\frac{M_p}{a^2}$$



$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta = 4M_p\vartheta$$

$$A = q4a\,\vartheta 2a = 8qa^2\vartheta$$

$$E = A \Rightarrow q = \frac{1}{2}\frac{M_p}{a^2}$$



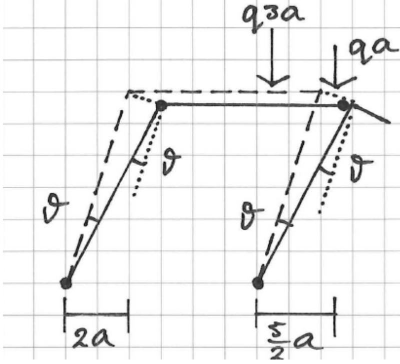
$$E = M_p\vartheta + M_p(\vartheta+\vartheta) + M_p(\vartheta+\vartheta) + {}$$

$$+ M_p\vartheta = 6M_p\vartheta$$

$$A = qa\,\vartheta\tfrac{5}{2}a + q3a\,\vartheta\tfrac{7}{2}a$$

$$= 13qa^2\vartheta$$

$$E = A \Rightarrow \boxed{q = \frac{6}{13}\frac{M_p}{a^2}}$$ decisive

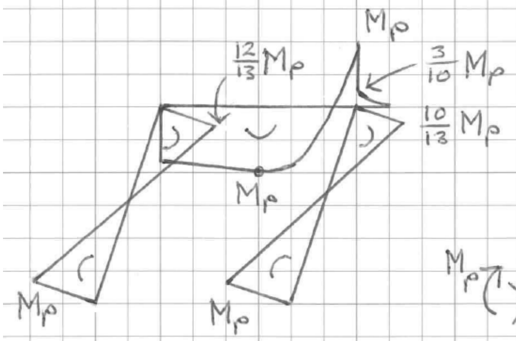


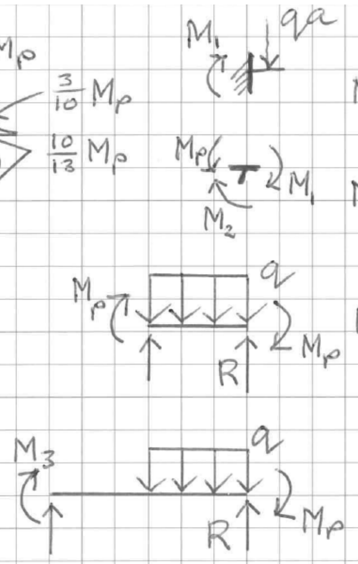
$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta = 4M_p\vartheta$$

$$A = q3a\,\vartheta 2a + qa\,\vartheta\tfrac{5}{2}a = \frac{17}{2}qa^2\vartheta$$

$$E = A \Rightarrow q = \frac{8}{17}\frac{M_p}{a^2}$$

Answer to Problem 1b





$$M_1 = qa\frac{1}{2}a = \frac{3}{13}M_p$$

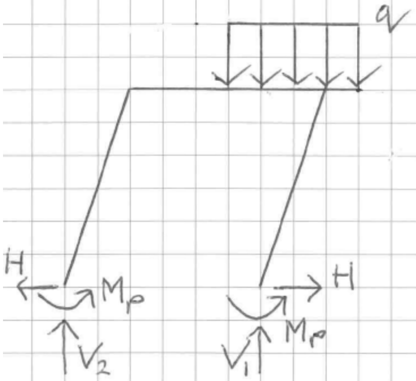
$$M_2 = M_p - M_1 = \frac{10}{13}M_p$$

$$M_p = R3a - M_p - q3a\frac{3}{2}a$$

$$\Rightarrow \quad R = \frac{53}{39}\frac{M_p}{a}$$

$$M_3 = R6a - M_p - q3a\frac{9}{2}a$$

$$= \frac{12}{13}M_p$$

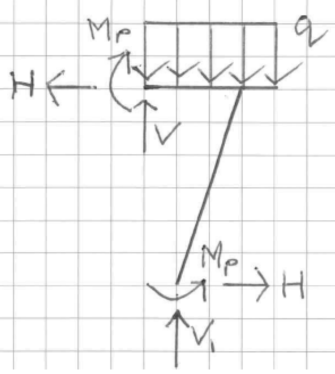


$$0 = V_1 6a + M_p - q4a7a + M_p \Rightarrow$$

$$V_1 = \frac{71}{39}\frac{M_p}{a}$$

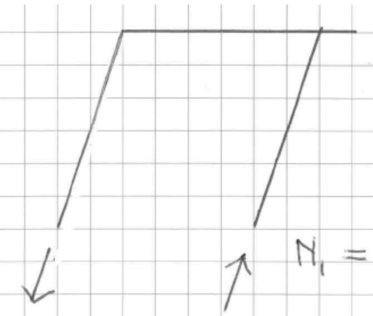
$$0 = q4a - V_1 - V_2 \Rightarrow$$

$$V_2 = \frac{1}{39}\frac{M_p}{a}$$



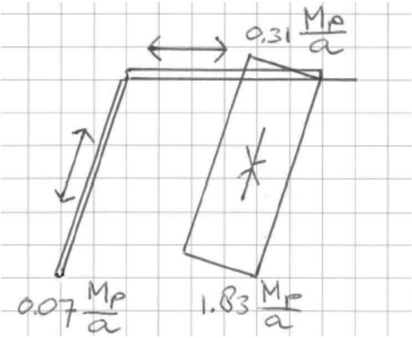
$$M_p = V_1 a + H6a + M_p - q4a2a \Rightarrow$$

$$H = \frac{73}{234}\frac{M_p}{a}$$



$$N_1 = H\frac{1}{\sqrt{10}} + V_1\frac{3}{\sqrt{10}} = 1.83\frac{M_p}{a}$$

$$N_2 = H\frac{1}{\sqrt{10}} - V_2\frac{3}{\sqrt{10}} = 0.07\frac{M_p}{a}$$

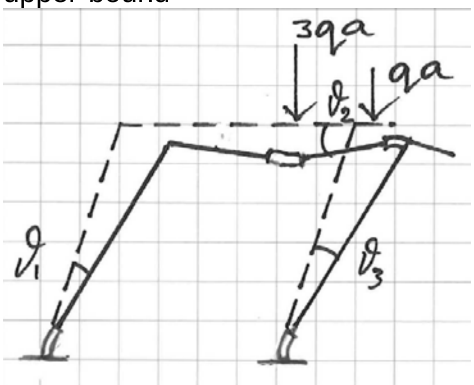


Answer to Problem 1c
lower-bound

```
> Np:=10.*Mp/a:
> s:=sqrt(10):
> eq1:= 0= (1-a1)*Np -V2*3/s -(1-a2)*Np/s:
> eq2:= 0= V1 - V2/s +(1-a2)*Np*3/s:
> eq3:= 0= a1*Mp -(1-a1)*Np*6*a -V1*5*a +a2*Mp:
> eq4:= 0= V1 -q*3*a +V4:
> eq5:= 0= q*3*a*3/2*a -V4*3*a +a1*Mp +a1*Mp:
> eq6:= 0= (1-a1)*Np +V3*3/s -(1-a3)*Np/s:
> eq7:= 0= V4 +q*a -(1-a3)*Np*3/s -V3/s:
> eq8:= 0= q*a*5/2*a -(1-a1)*Np*6*a +V4*2*a -a1*Mp -a3*Mp:
> solve({eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8},{a1,a2,a3,V1,V2,V3,V4,q});
```

$$\{V1 = \frac{0.01091410889\,Mp}{a},\ V2 = \frac{0.3046079498\,Mp}{a},\ V3 = \frac{0.2489320127\,Mp}{a},\ V4 = \frac{1.301921184\,Mp}{a},$$
$$a1 = 0.9682553065,\ a2 = 0.9909968498,\ a3 = 0.8249348610,\ q = \frac{0.4376117644\,Mp}{a^2}\}$$

upper-bound




```
> b:=10.: s:=sqrt(10):
> eq1:= t1*a/b/s +t1*6*a +(t1+t2)*a/b = -t3*a/b/s +t3*6*a -(t3+t2)*a/b:
> eq2:= -t1*a/b*3/s +t1*5*a = t3*a/b*3/s +t3*2*a +t2*3*a:
> E:= Mp*t1 +Mp*(t1+t2) +Mp*(t2+t3) +Mp*t3:
> A:= q*a*(t3*a/b*3/s +t3*5/2*a) +3*q*a*(t3*a/b*3/s +t3*2*a +t2*3/2*a):
> eq3:= E=A:
> solve({eq1,eq2,eq3},{t1,t3,q});
```

$$\{q = \frac{0.4376117644\,Mp}{a^2},\ t1 = 1.130731235\,t2,\ t3 = 1.215534913\,t2\}$$

Answer to Problem 2a
A, B, C, D, E, F

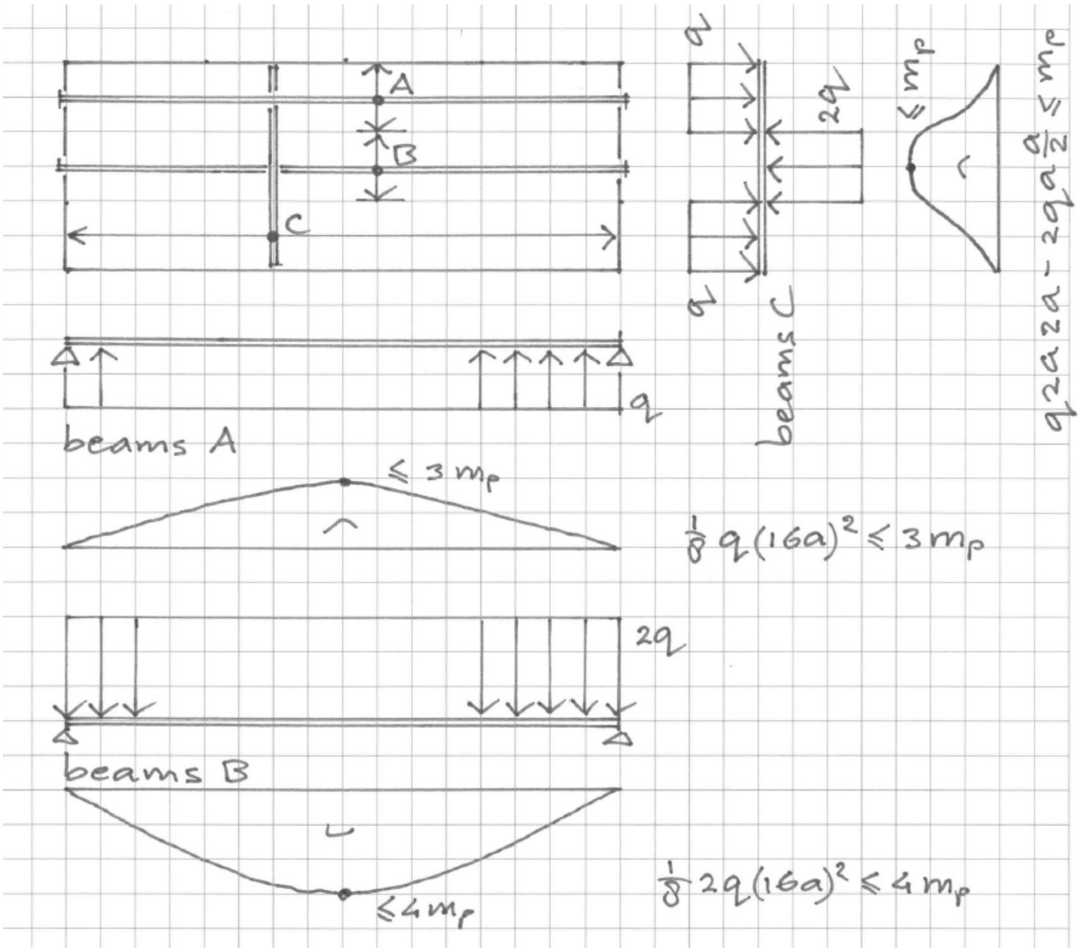
Answer to Problem 2b

$$E = 2\left[3m_p\, 4a \frac{2w}{4a} + m_p\, 4a \frac{2w}{4a}\right] = 16\, m_p w$$

$$A = q\, 2a\, 16a \frac{w}{2} = 16\, q a^2 w$$

$$E = A \Rightarrow q = \frac{m_p}{a^2}$$

Answer to Problem 2c



the solution to the equations is $q = \frac{1}{16}\frac{m_p}{a^2}$

Answer to Problem 3a
A

Answer to Problem 3b
B

Answer to Problem 3c
D

Answer to Problem 3d
A