Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CT4150 Plastic Analysis of Structures
Thursday 11 April 2013, 14:00 – 17:00 hours

Write your name and study number at the top right-hand of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

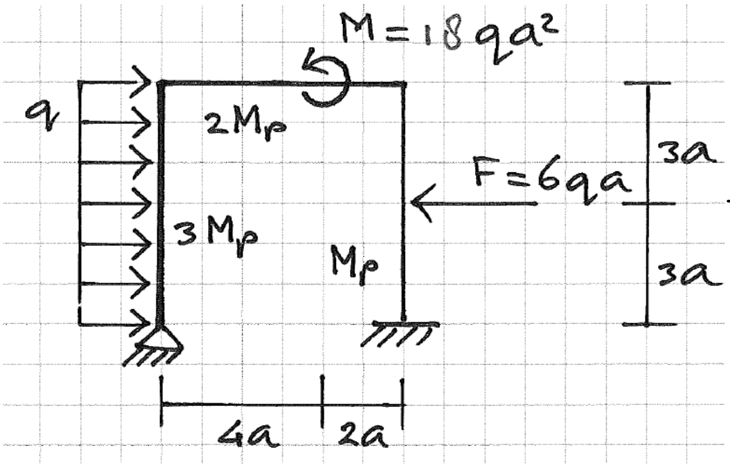

Figure 1. Frame structure

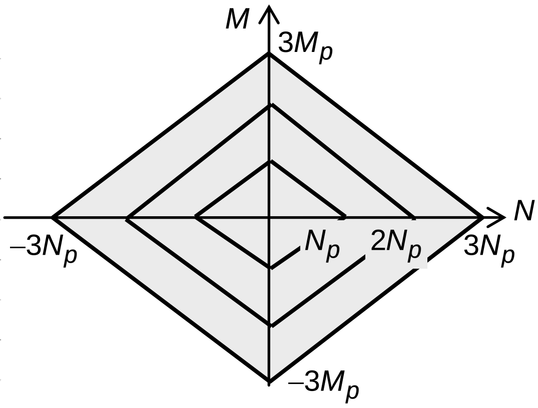

Figure 2. Yield contour

Problem 1

A frame consists of two columns and a beam (Fig.1). The columns have strengths $3M_p$ and M_p. The beam has a strength $2M_p$. The structure is loaded by a horizontal load *q*, a horizontal point load *F* and a moment *M*.

The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load *q* for all possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 12. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for *q* using Figure 3.
– Determine the smallest upper-bound for *q*.
You only need to write down the equations and not solve the equations (1.5 points).

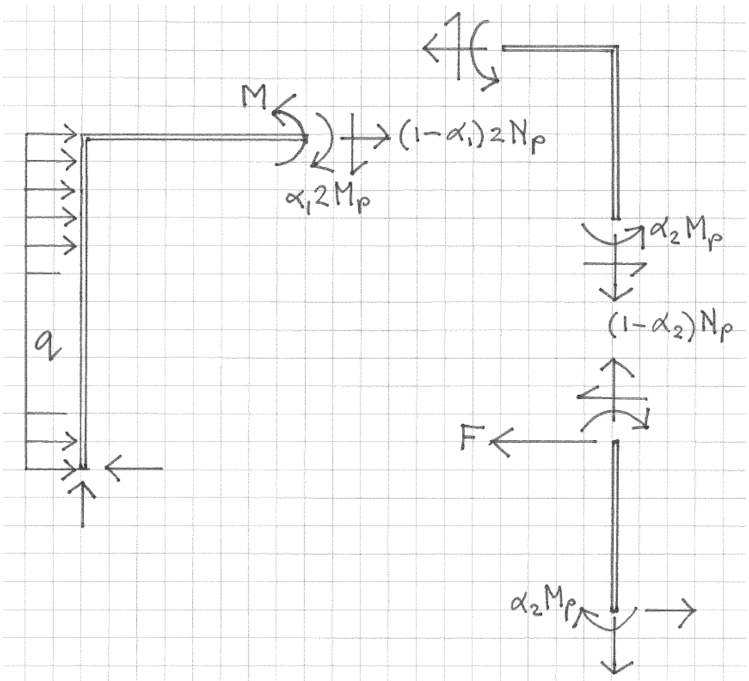

Figure 3. Forces and moments on frame members for a lower-bound analysis

Problem 2

A square reinforced concrete plate is simply supported at parts of its edges (Fig. 4). It carries an evenly distributed load *p*. The plate is homogeneous and orthotropic.

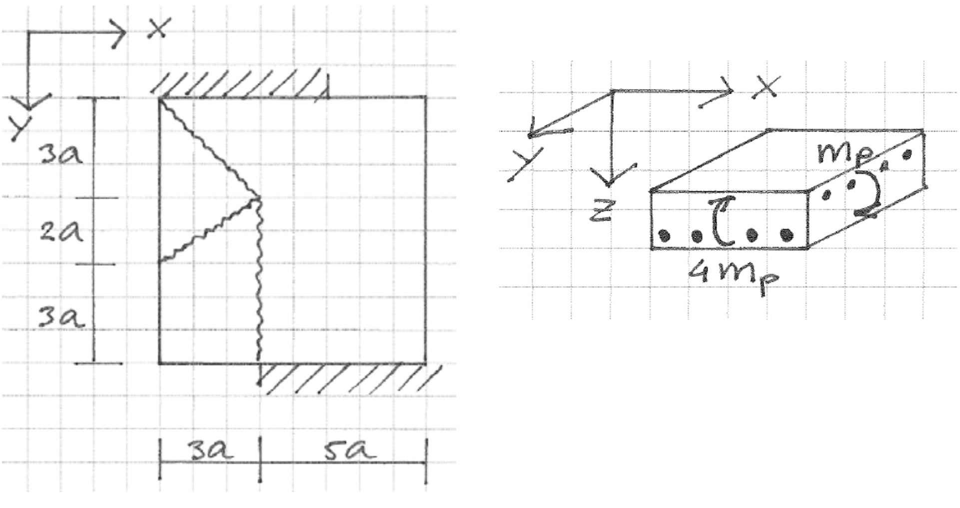

Figure 4. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

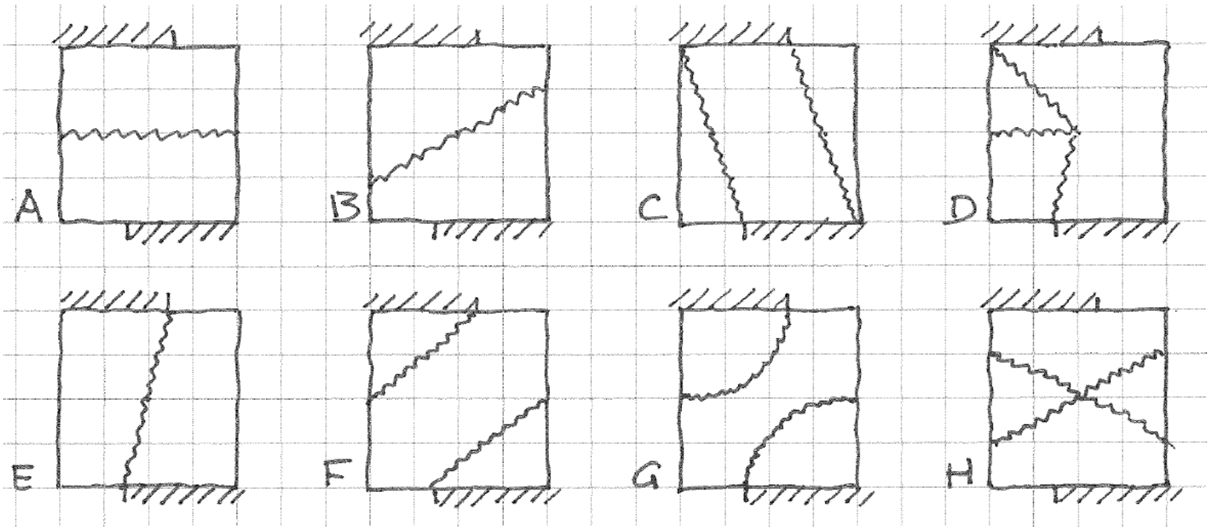

Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 4. Determine an <u>upper bound</u> for *p* expressed in m_p and *a* (1.5 point).

c Determine the largest <u>lower-bound</u> for *p* using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a The lowerbound theorem is important because ... Choose A, B, C or D (0.5 point).

A ... it gives a mathematical proof of the strength of structures.
B ... it provides the key to designing safe, economical and sustainable structures.
C ... it proves from an energy point of view that good structures can be designed.
D ... it gives a method for optimising structural strength.

b Reinforced concrete beams can display arch action and cable action. Steel beams can display cable action only. What causes this? Choose A, B, C or D (0.5 point).

A Reinforced concrete does not yield while steel does.
B Reinforced concrete beams are less sensitive to temperature than steel beams.
C In general, concrete plastic hinges expand while steel plastic hinges do not.
D Poisson's ratio of reinforced concrete is much smaller than that of steel.

c Consider a statically indetermined prestressed concrete beam. The prestressing is important for ... Choose from A to E. More than one can be selected (0.5 point).

A deflection
B crack width
C ductile bending failure
D brittle shear failure
E natural frequencies

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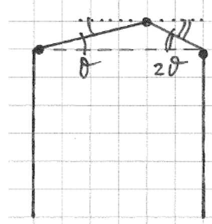
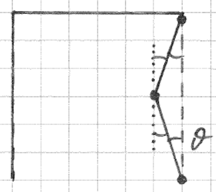
Answer to Problem 1a



$A = M\vartheta$

$E = 2M_p\vartheta + 2M_p(\vartheta + 2\vartheta) + M_p 2\vartheta$

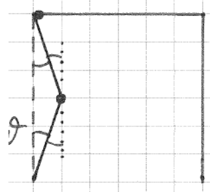
$A = E \Rightarrow q = \frac{5}{9}\frac{M_p}{a^2}$



$A = F\vartheta 3a$

$E = M_p\vartheta + M_p(\vartheta + \vartheta) + M_p\vartheta$

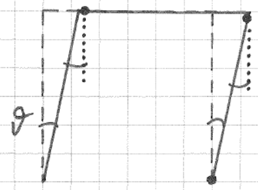
$A = E \Rightarrow \boxed{q = \frac{2}{9}\frac{M_p}{a^2}}$ decisive



$A = 2[q\,3a\,\frac{3}{2}a\,\vartheta]$

$E = 3M_p(\vartheta + \vartheta) + 2M_p\vartheta$

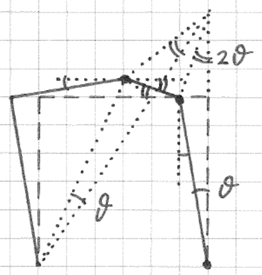
$A = E \Rightarrow q = \frac{8}{9}\frac{M_p}{a^2}$



$A = q6a3a\vartheta - F3a\vartheta = 0qa^2\vartheta$

$E = 2M_p\vartheta + M_p\vartheta + M_p\vartheta = 4M_p\vartheta$

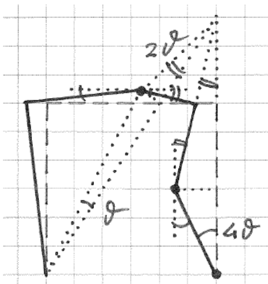
$A = E \Rightarrow q = \frac{4}{0}\frac{M_p}{a^2} = \infty$



$A = -q6a3a\vartheta + M\vartheta + F3a\vartheta$

$E = 2M_p(\vartheta + 2\vartheta) + M_p(2\vartheta + \vartheta) + M_p\vartheta$

$A = E \Rightarrow q = \frac{5}{9}\frac{M_p}{a^2}$



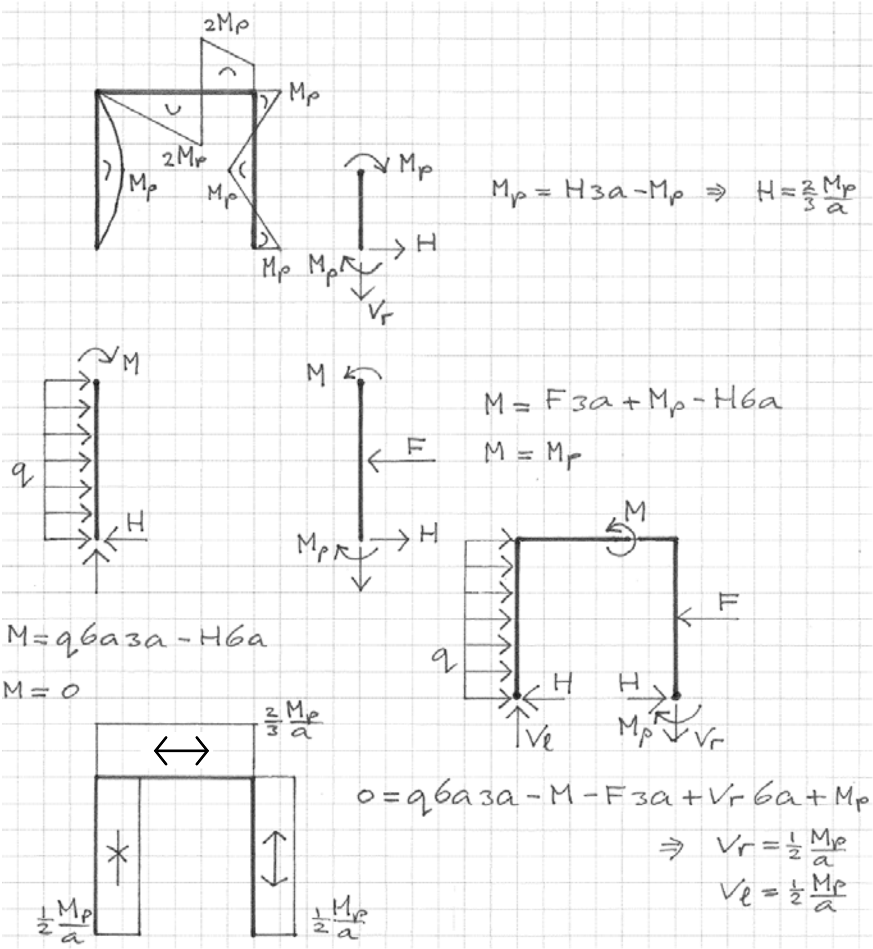
$A = -q6a3a\vartheta + M\vartheta + F3a4\vartheta$

$E = 2M_p(\vartheta + 2\vartheta) + M_p(2\vartheta + 4\vartheta) + M_p4\vartheta$

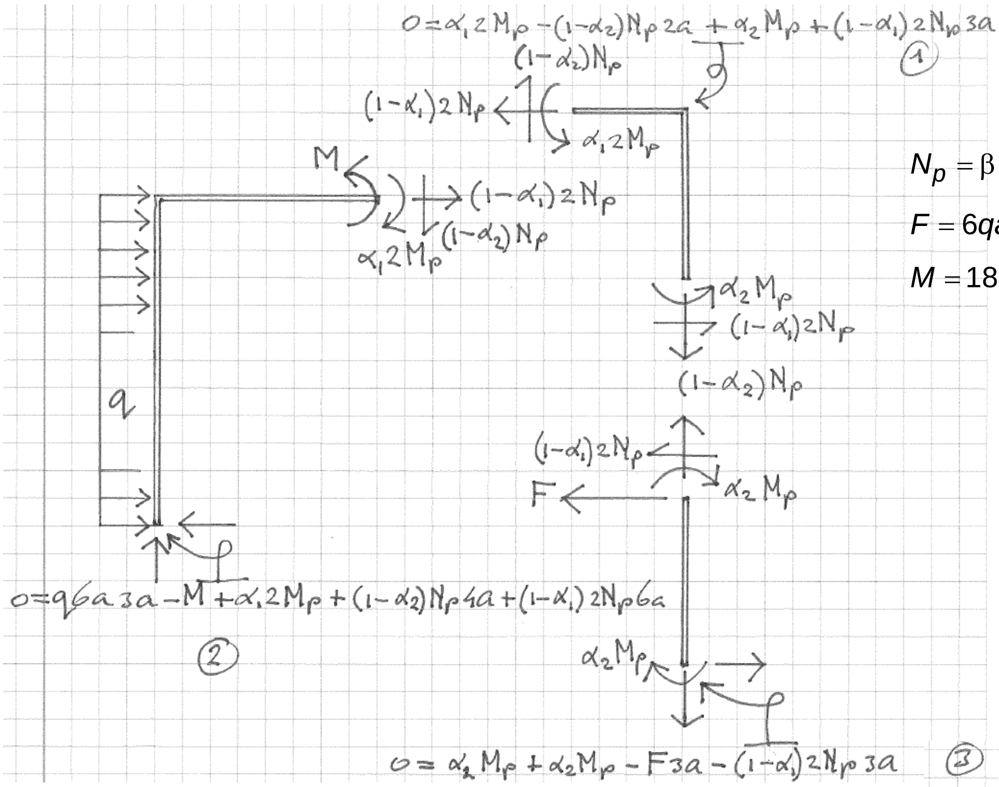
$E = A \Rightarrow \boxed{q = \frac{2}{9}\frac{M_p}{a^2}}$ decisive

0,222

Answer to Problem 1b



Answer to Problem 1c lower-bound



$N_p = \beta \frac{M_p}{a}$

$F = 6qa$

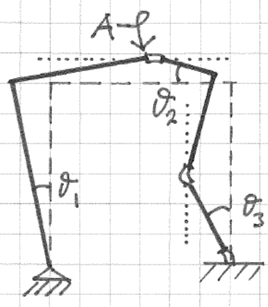
$M = 18qa^2$

3 equations : ① ② ③

3 unknown : q, α_1, α_2

The solution is : $q = \frac{756}{3455}\frac{M_p}{a^2}$, $\alpha_1 = \frac{3552}{3455}$, $\alpha_2 = \frac{3312}{3455}$

upper-bound



hor. disp. of A ←

$$\vartheta_1 6a = \vartheta_3 3a - \vartheta_2 3a + (\vartheta_1 + \vartheta_2)\frac{a}{\beta}$$

vert. disp. of A ↑

$$\vartheta_1 4a = \vartheta_3 \frac{a}{\beta} + (\vartheta_2 + \vartheta_3)\frac{a}{\beta} + \vartheta_2 2a$$

$$A = -q\, 6a\, 3a\, \vartheta_1 + M \vartheta_1 + F\, 3a\, \vartheta_3$$

$$E = 2M_p(\vartheta_1 + \vartheta_2) + M_p(\vartheta_2 + \vartheta_3) + M_p \vartheta_3$$

$$A = E$$

The solution to the equations is

$$q = \frac{756}{3455}\frac{M_p}{a^2},\ \vartheta_3 = \frac{691}{194}\vartheta_1,\ \vartheta_2 = \frac{793}{485}\vartheta_1$$

0,219

Answer to Problem 2a

A, C, D, F, H

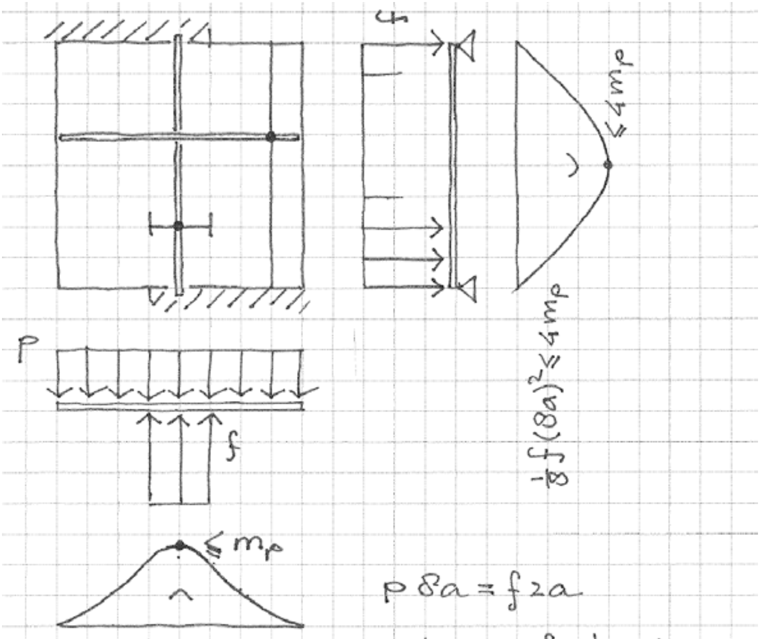
Answer to Problem 2b

$$A = p\, 3a\, 3a \frac{w}{2} + p \frac{1}{2} 3a\, 2a \frac{w}{3} + p \frac{1}{2} 3a\, 2a \frac{8}{15} w + $$
$$+ p \frac{1}{2} 3a\, 3a \frac{1}{5} w = 8 p a^2 w$$

$$E = m_p\, 3a \frac{1}{5}\frac{w}{a} + 4 m_p\, 3a \frac{w}{5a} + m_p\, 5a \frac{w}{3a} = \frac{70}{15} m_p w$$

$$A = E \Rightarrow p = \frac{7}{12}\frac{m_p}{a^2}$$

Answer to Problem 2c



$$p\,8a = f\,2a$$

$$p\,4a\,2a - f\,a\,\tfrac{1}{2}a \leq m_p$$

The solution to the equations is $p = \frac{1}{8}\frac{m_p}{a^2}$

Answer to Problem 3a
A

Answer to Problem 3b
C

Answer to Problem 3c
Any answer except C is right. If you included C than you get zero points for this answer.