Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CT4150 Plastic Analysis of Structures
Thursday 10 April 2014, 14:00 – 17:00 hours

Write your name and study number at the top right-hand of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

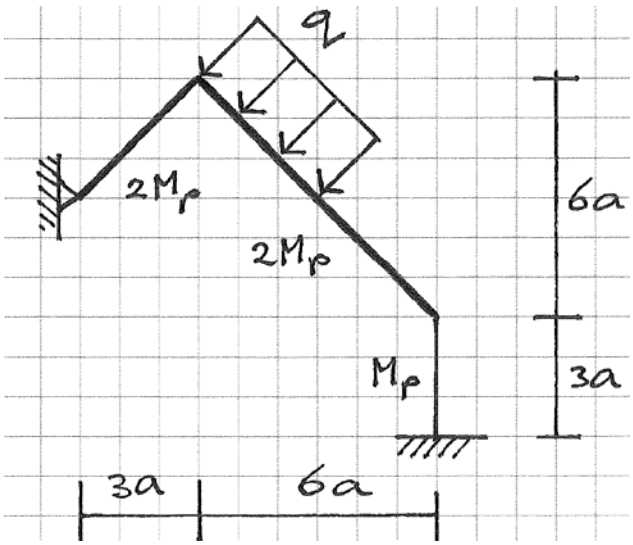

Figure 1. Frame structure

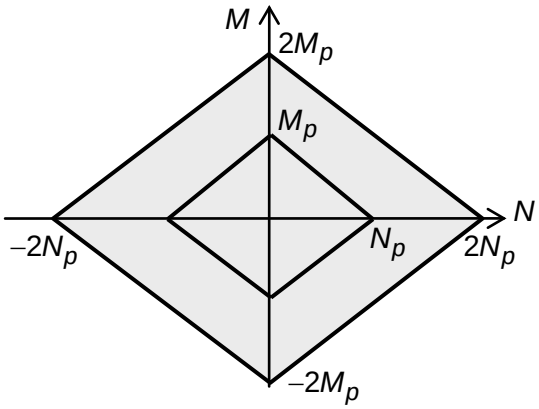

Figure 2. Yield contour

Problem 1

A frame consists of three members (Fig.1) Two members have a strength $2M_p$ the third has a strength M_p. All joints are rigid except for the left hand support, which is pinned. The structure is loaded by an inclined evenly distributed load q.

The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 63.2$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate is simply supported at some of its edges (Fig. 3). It carries an evenly distributed load *p*. The plate is homogeneous and orthotropic.

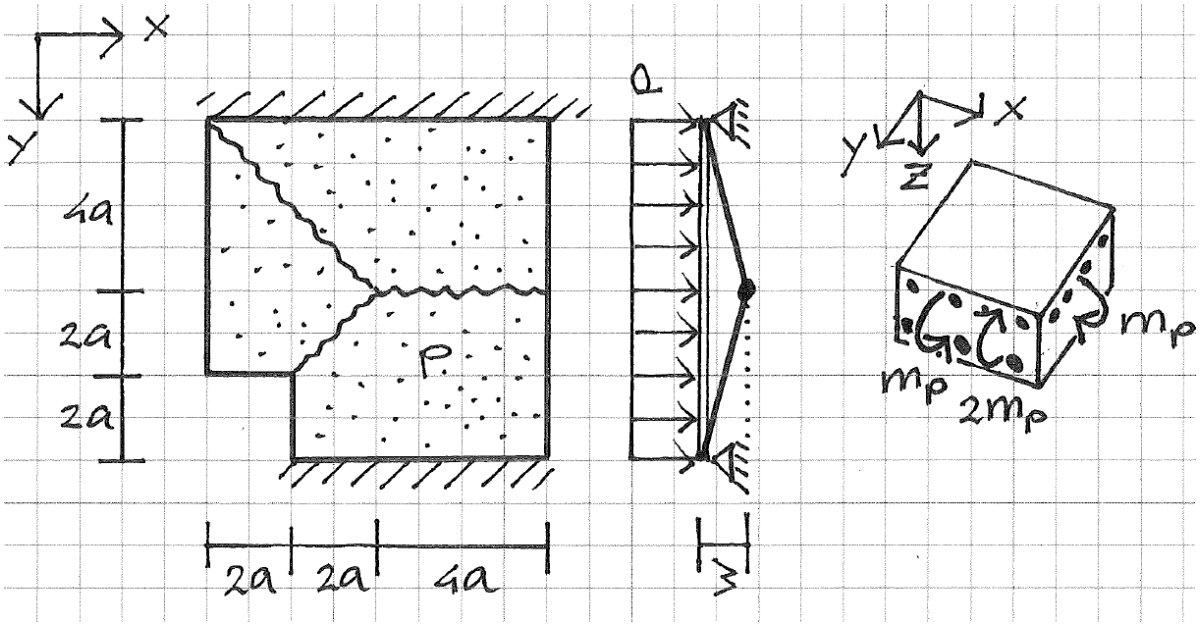

Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

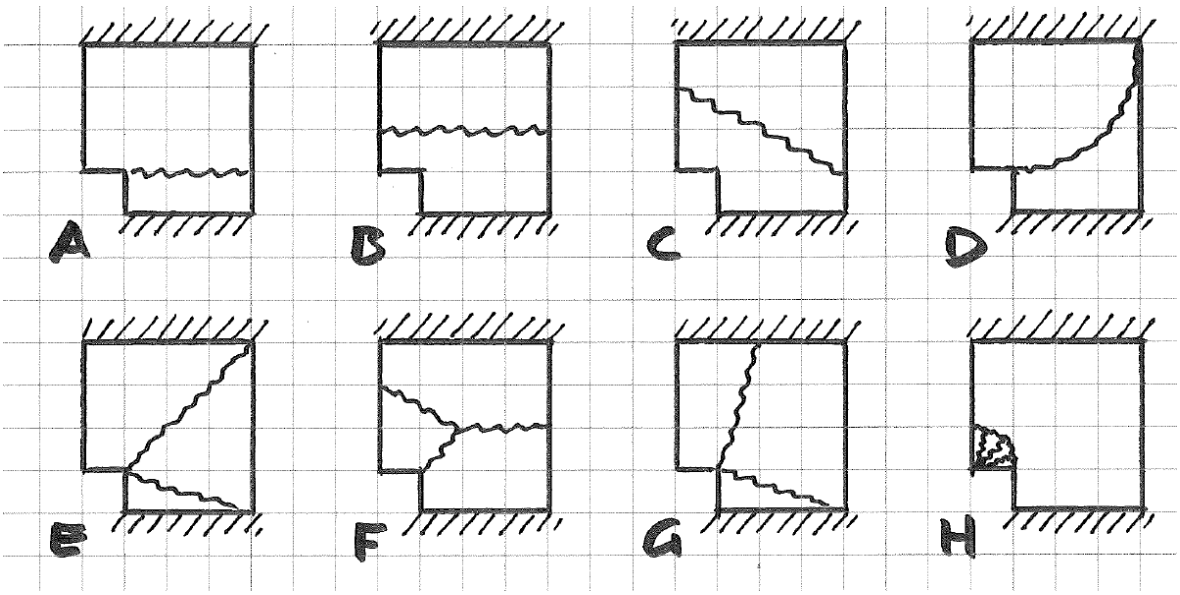

Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for *p* expressed in m_p and *a* (1.5 point).

c Determine the largest <u>lower-bound</u> for *p* using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Figure 5 shows a reinforced concrete floor loaded by a distributed load. The reinforcing bars are in parallel to the edges. We want to determine a lower-bound solution using torsion free beams. Is this possible? Choose A, B, C or D (0.5 point).

A Yes, torsion free beams are suitable for lower-bound analysis of any floor.
B Yes, a beam system can be designed that can carry the load on this floor to the supports in just bending without torsion.
C No, in this floor the load can only be carried by torsion.
D No, the yield lines would need to be in parallel to the reinforcing bars which is not possible in this floor.

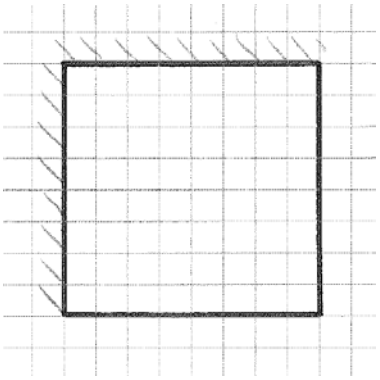

Figure 5. A square reinforced concrete floor simply supported at two edges

b Temperature loading can cause large stresses in statically indetermined structures. In structural analysis this load can be combined with other loads such as wind load and snow load. Does a temperature load reduce the strength of a ductile structure? Choose A, B, C or D (0.5 point).

A Yes; a temperature loading causes large stresses in a statically indetermined structure which can cause yielding of the material and failure of members or joints.
B Yes; the yielding caused by temperature loading can damage a structure significantly. The climate can worsen this damage, which in time can lead to collapse.
C No; temperature deformation is absorbed in the plastic deformation of a “real” load.
D No; the temperature stresses are in equilibrium. They can be added to the stresses by other loads which are also in equilibrium and the structure will still fulfil the lower-bound theorem.

c Sometimes we check a structural design and conclude that it cannot be built because it would not have sufficient strength. Nonetheless, the structure has been build. Subsequently, we try to show that it is has sufficient strength after all. Do we use elastic or plastic analyses for this? Choose A, B, C or D (0.5 point).

A Elastic analysis because it provides a lower bound to the strength of the structure.
B Elastic analysis because it is more suitable for designing.
C Plastic analysis because it can predict the ultimate load on a structure.
D Plastic analysis because it can describe the serviceability limit state of the structure more accurately than an elastic analysis can.

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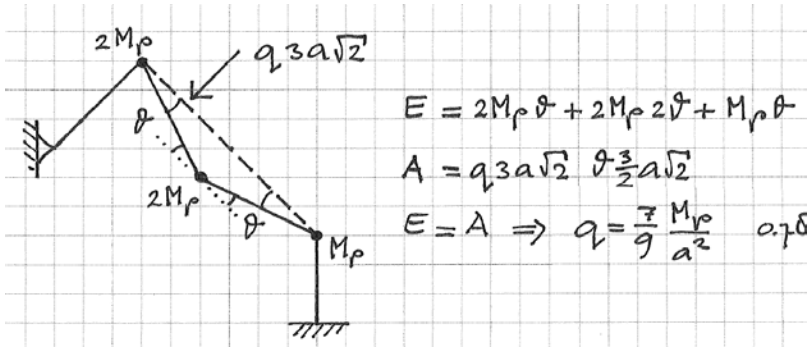
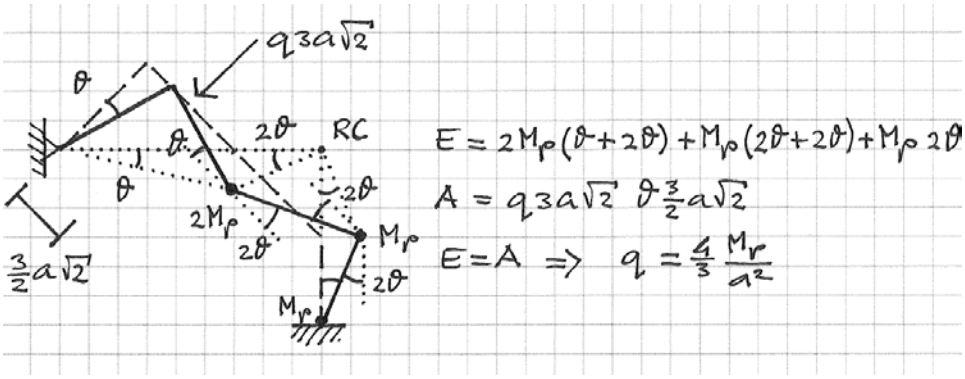
Answer to Problem 1a



$$E = 2M_p\vartheta + 2M_p 2\vartheta + M_p\vartheta$$

$$A = q\,3a\sqrt{2}\;\vartheta\tfrac{3}{2}a\sqrt{2}$$

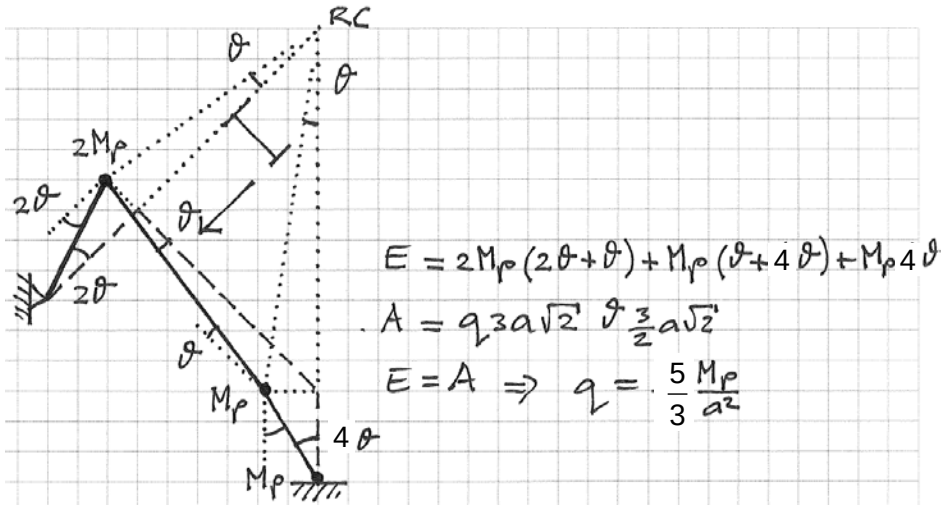
$$E = A \Rightarrow q = \frac{7}{9}\frac{M_p}{a^2} \qquad 0.78$$



$$E = 2M_p(\vartheta+2\vartheta) + M_p(2\vartheta+2\vartheta) + M_p 2\vartheta$$

$$A = q\,3a\sqrt{2}\;\vartheta\tfrac{3}{2}a\sqrt{2}$$

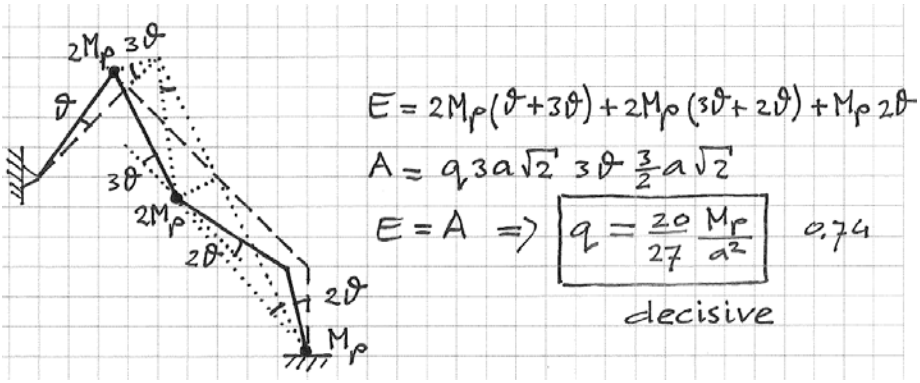
$$E = A \Rightarrow q = \frac{4}{3}\frac{M_p}{a^2}$$



$$E = 2M_p(2\vartheta+\vartheta) + M_p(\vartheta+4\vartheta) + M_p 4\vartheta$$

$$A = q\,3a\sqrt{2}\;\vartheta\tfrac{3}{2}a\sqrt{2}$$

$$E = A \Rightarrow q = \frac{5}{3}\frac{M_p}{a^2}$$



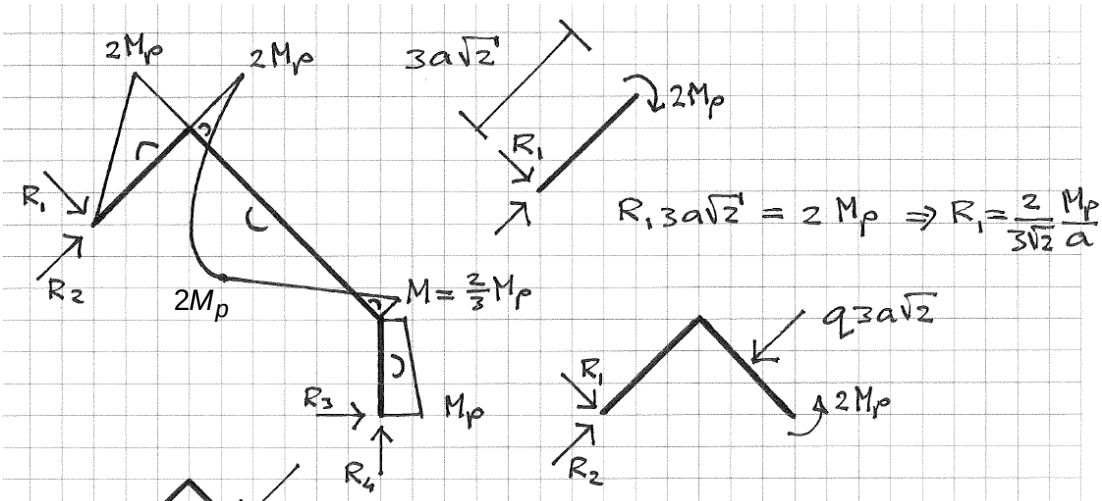
$$E = 2M_p(\vartheta+3\vartheta) + 2M_p(3\vartheta+2\vartheta) + M_p 2\vartheta$$

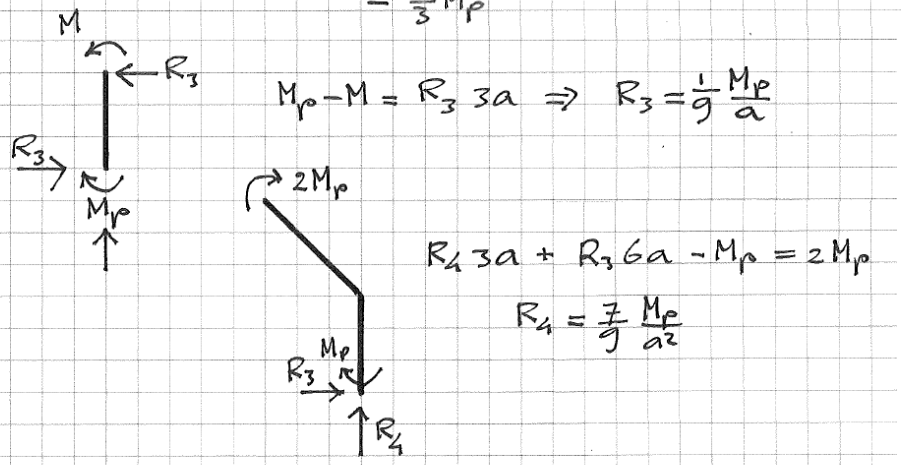
$$A = q\,3a\sqrt{2}\;3\vartheta\tfrac{3}{2}a\sqrt{2}$$

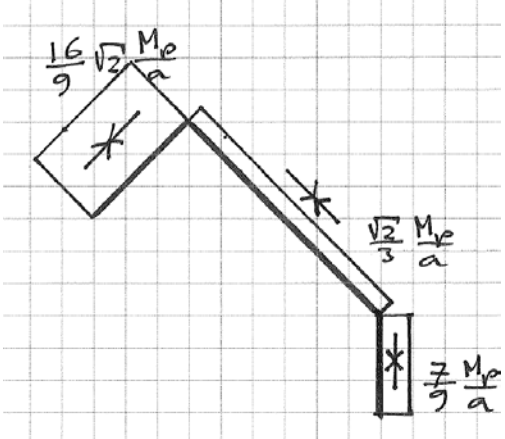
$$E = A \Rightarrow \boxed{q = \frac{20}{27}\frac{M_p}{a^2}} \qquad 0.74$$

decisive

Answer to Problem 1b



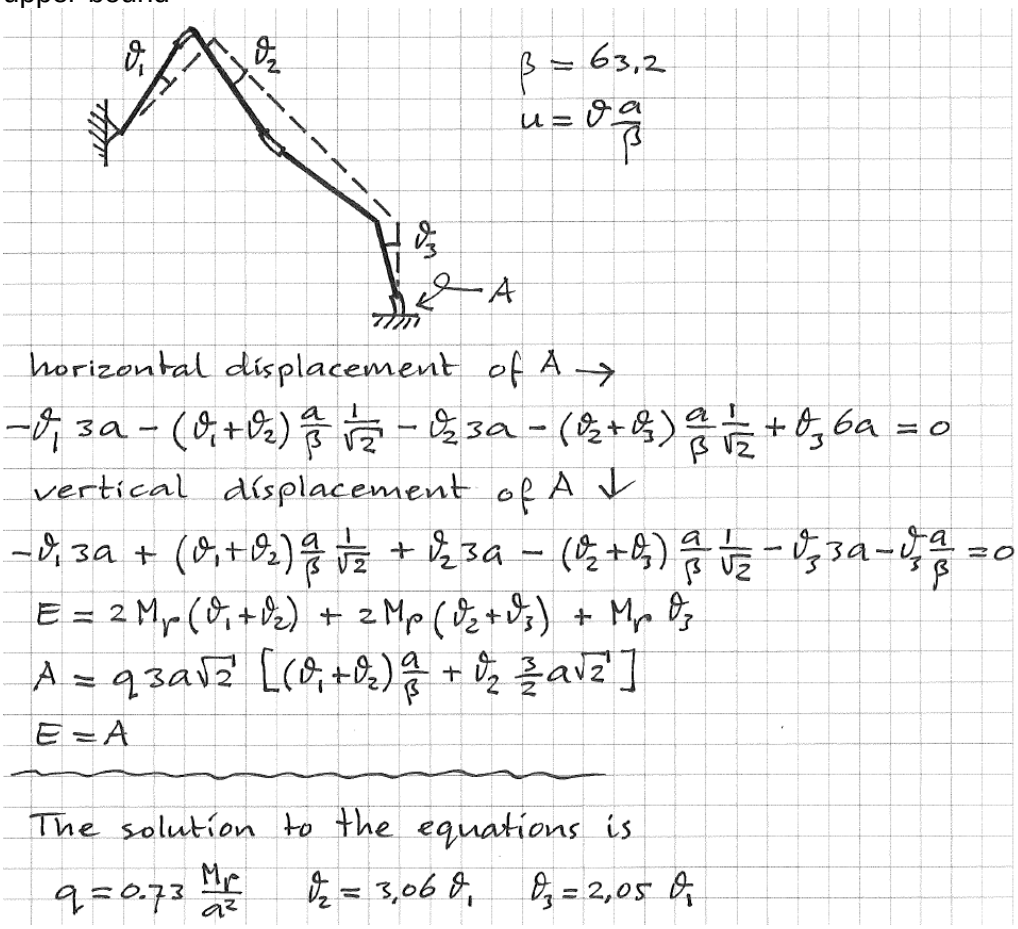




Answer to Problem 1c

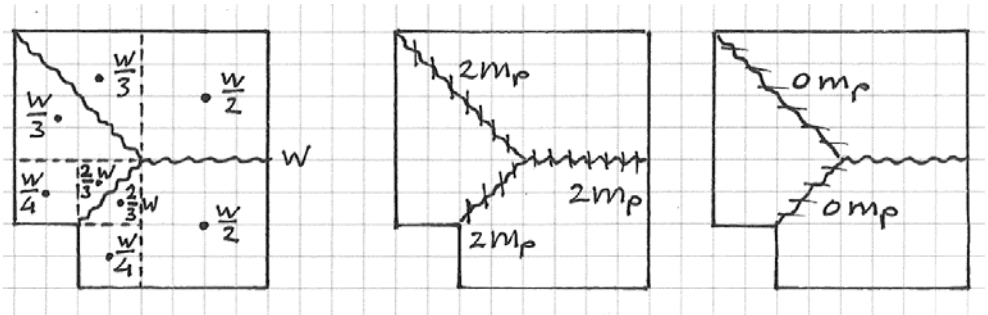
lower-bound

upper-bound



Answer to Problem 2a
A, B, E, F, H

Answer to Problem 2b



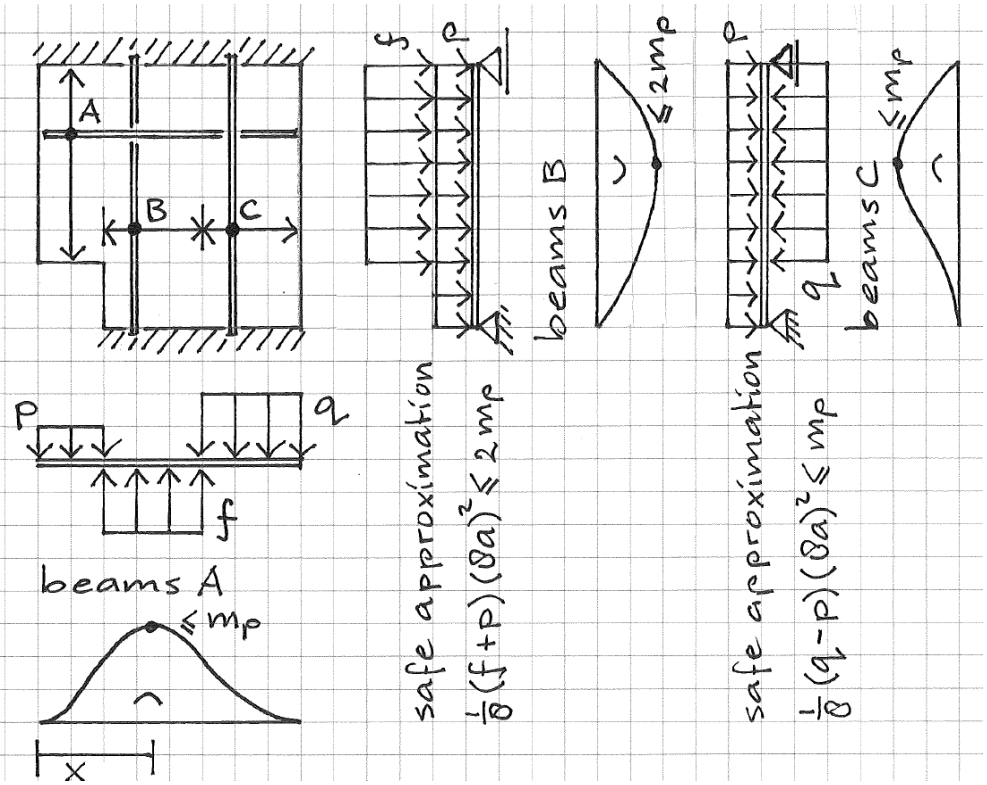
$$E = 2m_p\left(\frac{w}{4a}+\frac{w}{4a}\right)4a + 2m_p\frac{w}{4a}2a + 2m_p\frac{w}{4a}4a = 7m_p w$$

$$A = 2\left(p\,4a\,4a\,\frac{w}{2}\right) + 2\left(p\,\frac{1}{2}4a\,4a\,\frac{w}{3}\right) + 2\left(p\,2a\,2a\,\frac{w}{4}\right) + 2\left(p\,\frac{1}{2}2a\,2a\,\frac{2}{3}w\right)$$

$$= 26\,p a^2 w$$

$$E = A \Rightarrow \quad p = \frac{7}{26}\frac{m_p}{a^2} \qquad 0.27$$

Answer to Problem 2c



$$p\,2a - f\,3a + q\,3a = 0$$

$$p\,2a\,a - f\,3a\,\frac{7}{2}a + q\,3a\,\frac{13}{2}a = 0$$

$$p\,2a - (x-2a)f = 0$$

$$p\,2a\,(x-a) - f(x-2a)\frac{x-2a}{2} \le m_p$$

The largest solution to the equations is

$$p = \frac{9}{80}\frac{m_p}{a^2} \qquad 0.11$$

Answer to Problem 3a
C
B is somewhat right because beams can be designed in the diagonal directions. The problem that we do not know the strength of these beams can be solved by using the yield contour of reinforced concrete plates. (B = 0.2 point; A = 0.1 point)

Answer to Problem 3b
C

Answer to Problem 3c
C