Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CT4150 Plastic Analysis of Structures
Thursday 23 January 2014, 14:00 – 17:00 hours

Write your name and study number at the top right-hand of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

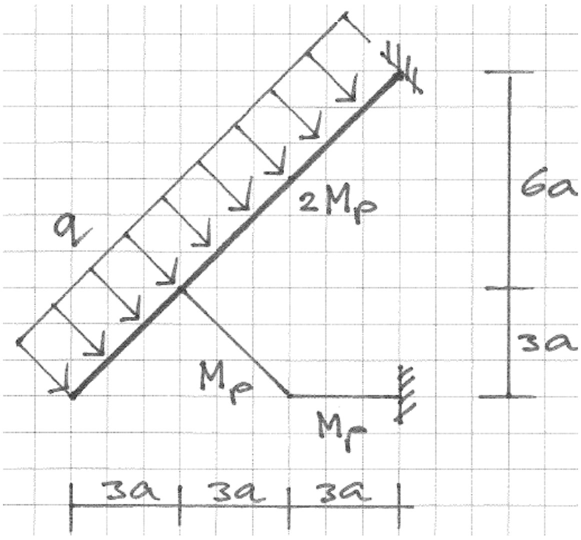

Figure 1. Frame structure

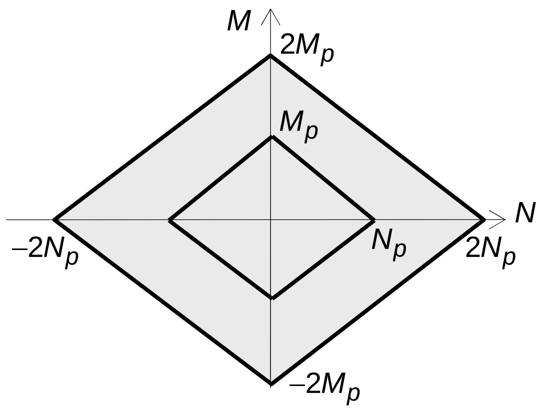

Figure 2. Yield contour

Problem 1

A frame consists of three members (Fig.1) Two members have a strength M_p the third has a strength $2M_p$. All joints are rigid. The structure is loaded by an inclined evenly distributed load q.

The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 50.1. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate is fixed and simply supported at some of its edges (Fig. 3). It carries an evenly distributed load *p*. The plate is homogeneous and orthotropic.

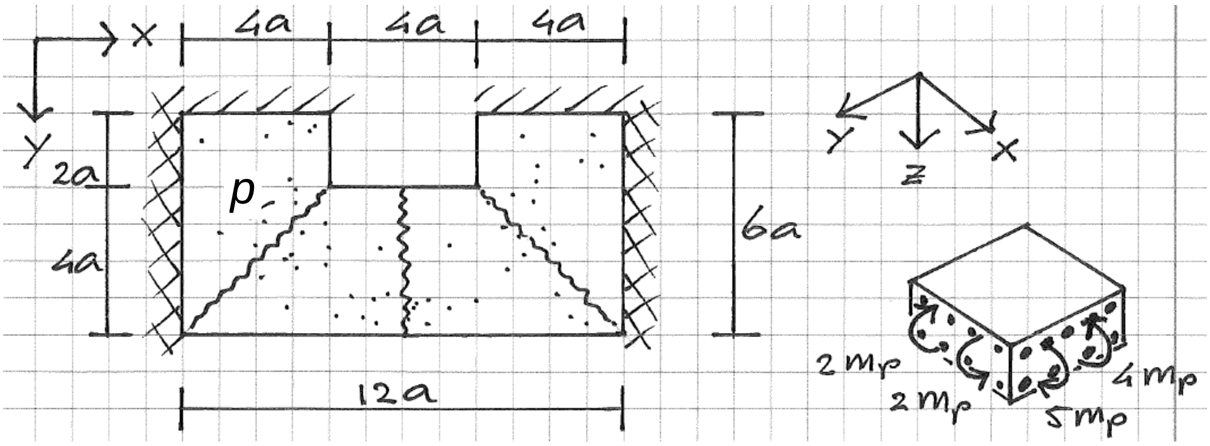

Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

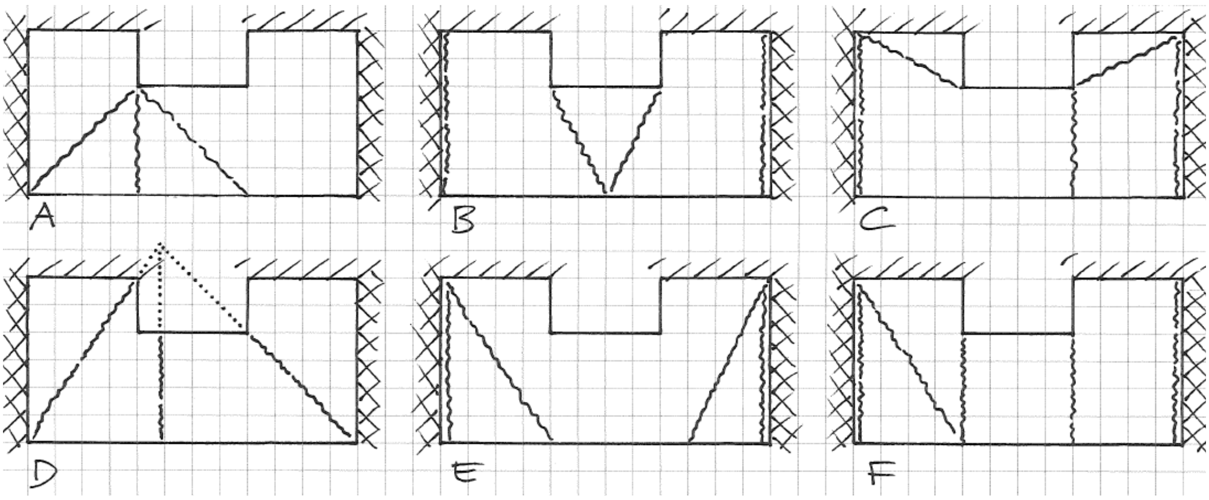

Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for *p* expressed in m_p and *a* (1.5 point).

c Determine the largest <u>lower-bound</u> for *p* using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Most engineers use elastic analysis for determining the force flow in a structure and plastic analysis for determining cross-section capacity. Why do they not use plastic analysis for both? Choose A, B, C or D (0.5 point).

A They have not been educated in plastic analysis of the whole structure.
B Elastic analysis is always safe compared to plastic analysis.
C Elastic analysis is faster than plastic analysis because software is used.
D Elastic analysis is more convenient in design than plastic analysis. It shows which dimensions are necessary and sufficient.

b Steel structures are commonly assembled out of accurately shaped members. However, these members will not fit perfectly and some force (a very large hammer) is needed to make them fit. Do misfits reduce the strength of ductile structures? Choose A, B, C or D (0.5 point).

A Yes; a misfit causes large stresses in a statically indetermined structure which can cause yielding of the material and failure of members or joints.
B Yes; using a hammer will damage the structure significantly which can lead to failure.
C No; the deformation of misfits will be absorbed in the plastic deformation of a “real” load.
D No; the stresses due to a misfit are in equilibrium. They can be added to the stresses by other loads which are also in equilibrium and the structure will still fulfil the lowerbound theorem.

c Sometimes we check a structural design and conclude that it cannot be built because it would not have sufficient strength. Nonetheless, the structure has been build. Subsequently, we try to show that it is strong enough after all. The Dutch word for the latter is “heelrekenen”. Propose an English translation of this word (0.5 point).

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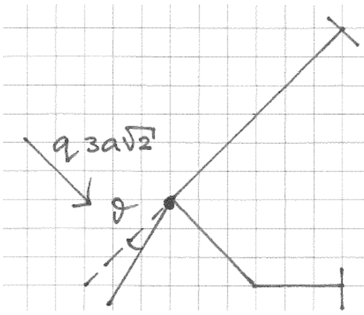
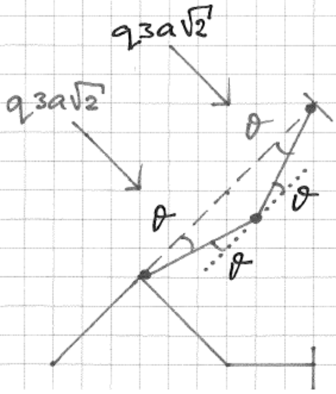
Answer to Problem 1a



$E = 2M_p\vartheta$

$A = q\,3a\sqrt{2}\,\vartheta\,\tfrac{1}{2}\,3a\sqrt{2}$

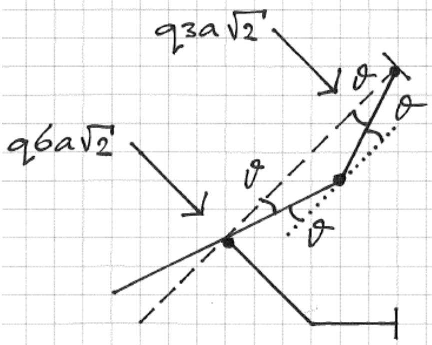
$E = A \Rightarrow q = \frac{2}{9}\frac{M_p}{a^2}$



$E = 2M_p\vartheta + 2M_p 2\vartheta + 2M_p\vartheta$

$A = 2[q\,3a\sqrt{2}\,\vartheta\,\tfrac{1}{2}\,3a\sqrt{2}]$

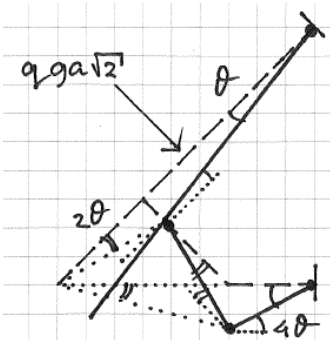
$E = A \Rightarrow q = \frac{4}{9}\frac{M_p}{a^2}$



$E = 2M_p\vartheta + 2M_p 2\vartheta + M_p\vartheta$

$A = q\,3a\sqrt{2}\,\vartheta\,\tfrac{1}{2}\,3a\sqrt{2}$

$E = A \Rightarrow q = \frac{7}{9}\frac{M_p}{a^2}$

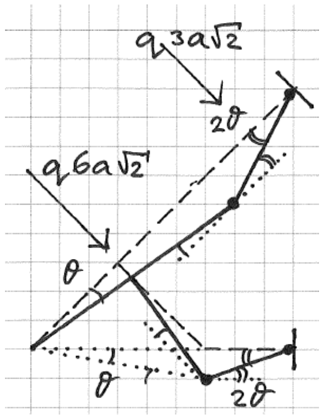


$E = 2M_p\vartheta + M_p(\vartheta + 2\vartheta) + M_p(2\vartheta + 4\vartheta) + M_p 4\vartheta$

$A = q\,9a\sqrt{2}\,\vartheta\,\tfrac{1}{2}\,9a\sqrt{2}$

$E = A \Rightarrow \boxed{q = \frac{15}{81}\frac{M_p}{a^2}}$ decisive

0.185

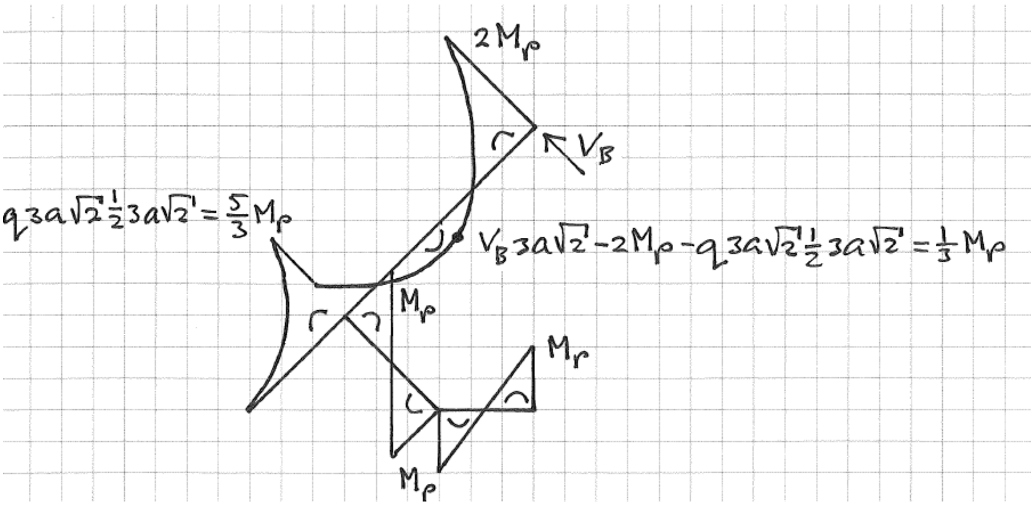


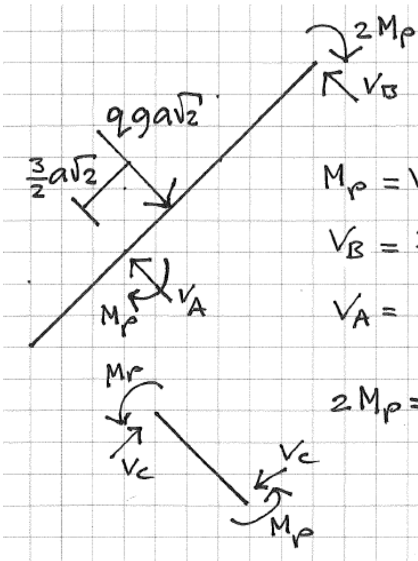
$$E = 2M_p\,2\vartheta + 2M_p(2\vartheta+\vartheta) + M_p(\vartheta+2\vartheta) + M_p\,2\vartheta$$

$$A = q\,3a\sqrt{2}\,2\vartheta\,\tfrac{1}{2}\,3a\sqrt{2} + q\,6a\sqrt{2}\,\vartheta\,3a\sqrt{2}$$

$$E = A \Rightarrow \quad q = \frac{5}{18}\frac{M_p}{a^2}$$

Answer to Problem 1b



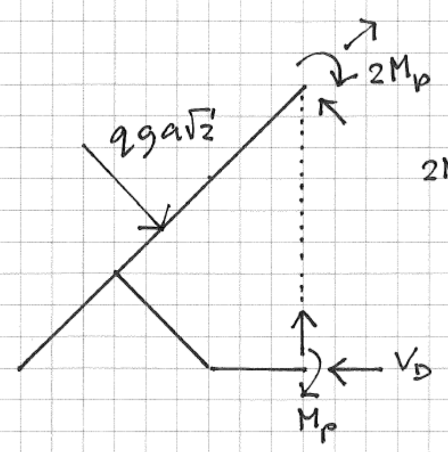


$$M_p = V_B\,6a\sqrt{2} - 2M_p - q\,9a\sqrt{2}\,\tfrac{3}{2}a\sqrt{2} \Rightarrow$$

$$V_B = \frac{2}{3}\sqrt{2}\,\frac{M_p}{a}$$

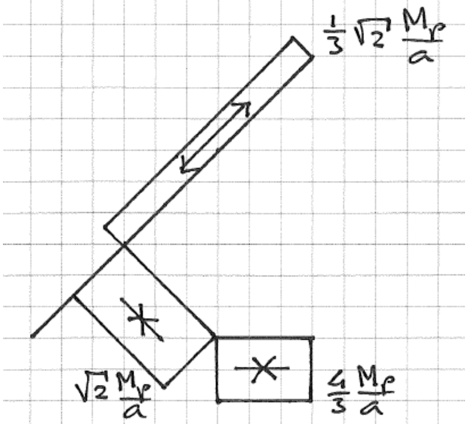
$$V_A = q\,9a\sqrt{2} - V_B = \sqrt{2}\,\frac{M_p}{a}$$

$$2M_p = V_C\,3a\sqrt{2} \Rightarrow V_C = \frac{1}{3}\sqrt{2}\,\frac{M_p}{a}$$

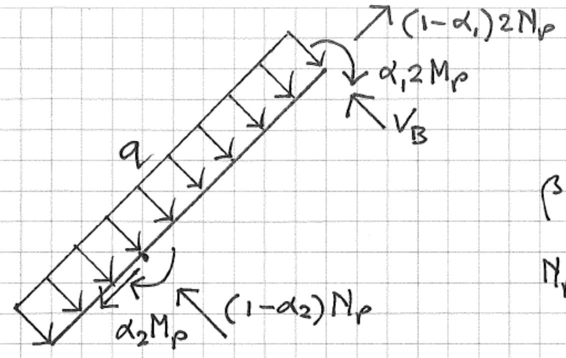


$$2M_p = q9a\sqrt{2}\,\tfrac{1}{2}9a\sqrt{2} - M_p - V_D\,9a \Rightarrow$$

$$V_D = \tfrac{4}{3}\frac{M_p}{a}$$



Answer to Problem 1c lower-bound



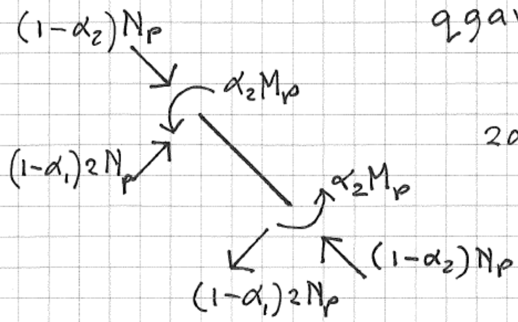
$$\beta = 50.1$$

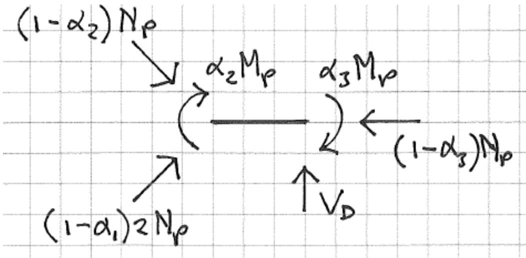
$$N_p = \beta\frac{M_p}{a}$$

$$\alpha_1 2M_p = q9a\sqrt{2}\,\tfrac{1}{2}9a\sqrt{2} - \alpha_2 M_p - (1-\alpha_2)N_p\,6a\sqrt{2} \quad (1)$$

$$q9a\sqrt{2} = V_B + (1-\alpha_2)N_p \quad (2)$$

$$2\alpha_2 M_p = (1-\alpha_1)2N_p\,3a\sqrt{2} \quad (3)$$





$$(1-\alpha_2)N_p\frac{1}{\sqrt{2}} + (1-\alpha_1)2N_p\frac{1}{\sqrt{2}} = (1-\alpha_3)N_p \quad (4)$$

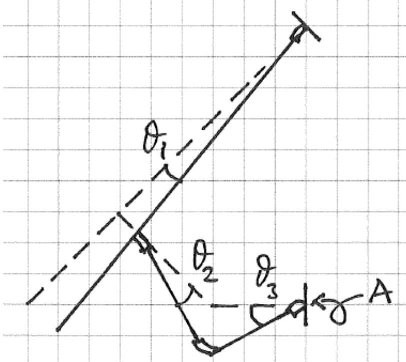
$$(1-\alpha_2)N_p\frac{1}{\sqrt{2}} - (1-\alpha_1)2N_p\frac{1}{\sqrt{2}} = V_D \quad (5)$$

$$\alpha_2 M_p + \alpha_3 M_p = V_D\, 3a \quad (6)$$

6 equations

6 unknown: α_1 α_2 α_3 V_B V_D q

upper-bound



$\beta = 50.1$

$u = \vartheta\frac{a}{\beta}$

horizontal displacement of A $\rightarrow$

$$-\vartheta_1\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_1 6a - (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} - \vartheta_2 3a - (\vartheta_2+\vartheta_3)\frac{a}{\beta}\frac{1}{\sqrt{2}} - \vartheta_3\frac{a}{3\beta} = 0$$

vertical displacement of A $\downarrow$

$$\vartheta_1\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_1 6a - (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_2 3a - (\vartheta_2+\vartheta_3)\frac{a}{\beta}\frac{1}{\sqrt{2}} - \vartheta_3 3a = 0$$

$$E = 2M_p\vartheta_1 + M_p(\vartheta_1+\vartheta_2) + M_p(\vartheta_2+\vartheta_3) + M_p\vartheta_3$$

$$A = q\, 9a\sqrt{2}\,\vartheta_1\frac{1}{2}9a\sqrt{2}$$

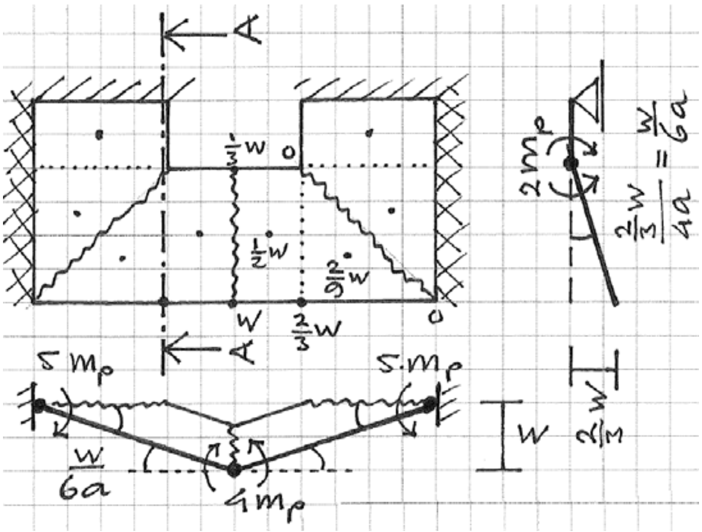
$$E = A$$

The solution to the equations is

$\vartheta_2 = 1.93\,\vartheta_1$ $\quad\vartheta_3 = 3.89\,\vartheta_1$ $\quad q = 0.181\frac{M_p}{a^2}$

Answer to Problem 2a
A, C, D, E

Answer to Problem 2b

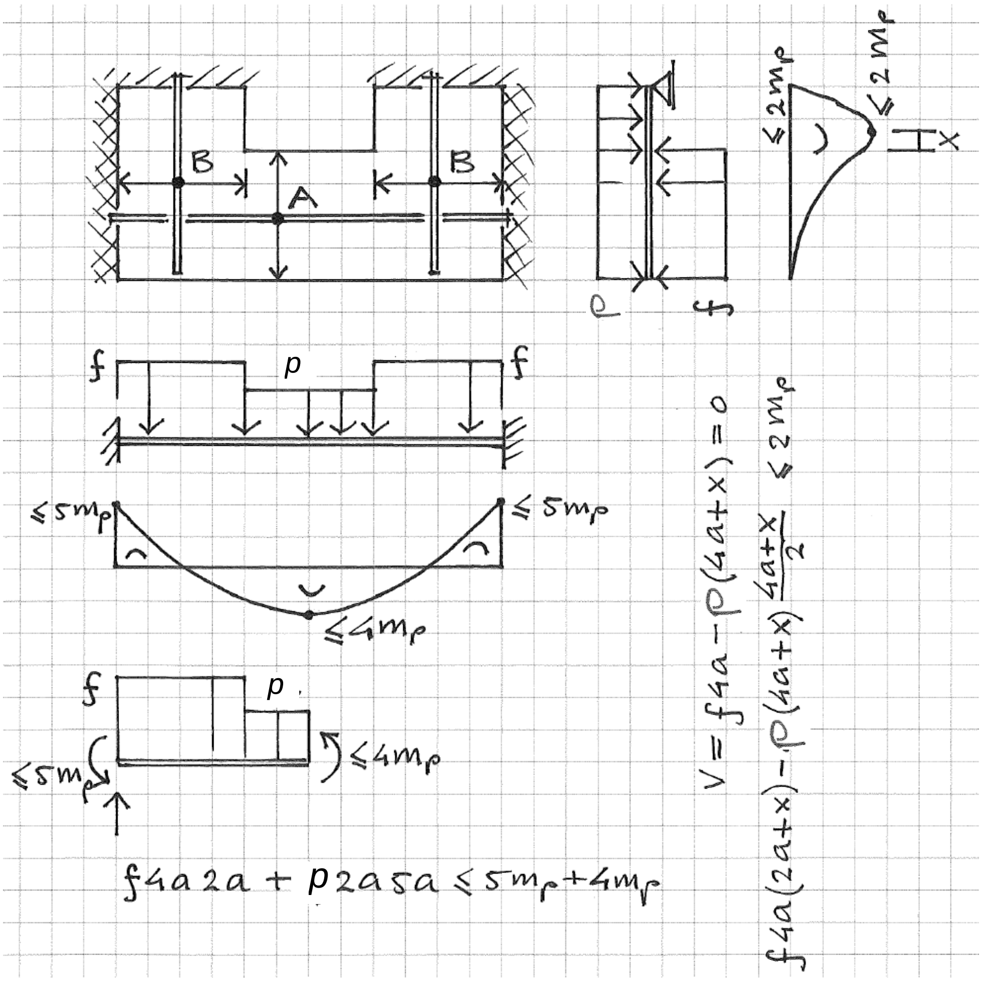


$$E = 4m_p 4a\left(\frac{w}{6a} + \frac{w}{6a}\right) + 2\left[5m_p 4a\frac{w}{6a} + 2m_p 4a\frac{w}{6a}\right] = \frac{44}{3}m_p w$$

$$A = 2\left[p\,2a\,4a\,\tfrac{1}{2}w + p\,\tfrac{1}{2}\,4a\,4a\,\tfrac{2}{9}w\right] = \frac{104}{9}pa^2w$$

$$E = A \Rightarrow \quad p = \frac{33}{26}\,\frac{m_p}{a^2}$$

Answer to Problem 2c



Answer to Problem 3a
D

Answer to Problem 3b
C

Answer to Problem 3c
Any reasonable word has been rewarded with 0.5 point.
17 different words have been proposed.
Possibly the best word is “mending math” by Kalle Nijs.