Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 9 April 2015, 14:00 – 17:00 hours

Write your name and study number at the top right-hand of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

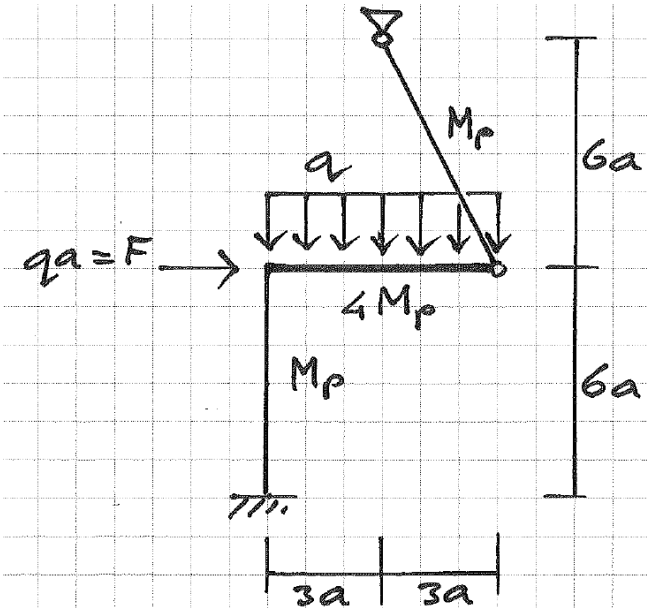

Figure 1. Frame structure

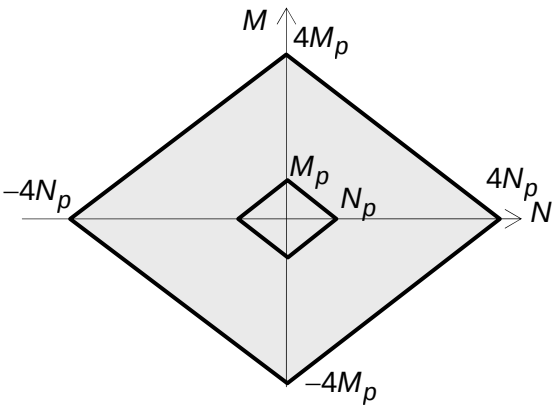

Figure 2. Yield contours

Problem 1

A frame consists of a column, a beam and an inclined cable (Fig.1) The column has a strength M_p, the beam has a strength $4M_p$ and the cable has strength M_p. The column is rigidly connected to the foundation and to the beam. The structure is loaded by an evenly distributed load q and a horizontal load F. The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 4$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has fixed and simply supported edges (Fig. 3). It carries an evenly distributed load *p* over part of the plate. The plate is homogeneous and orthotropic.

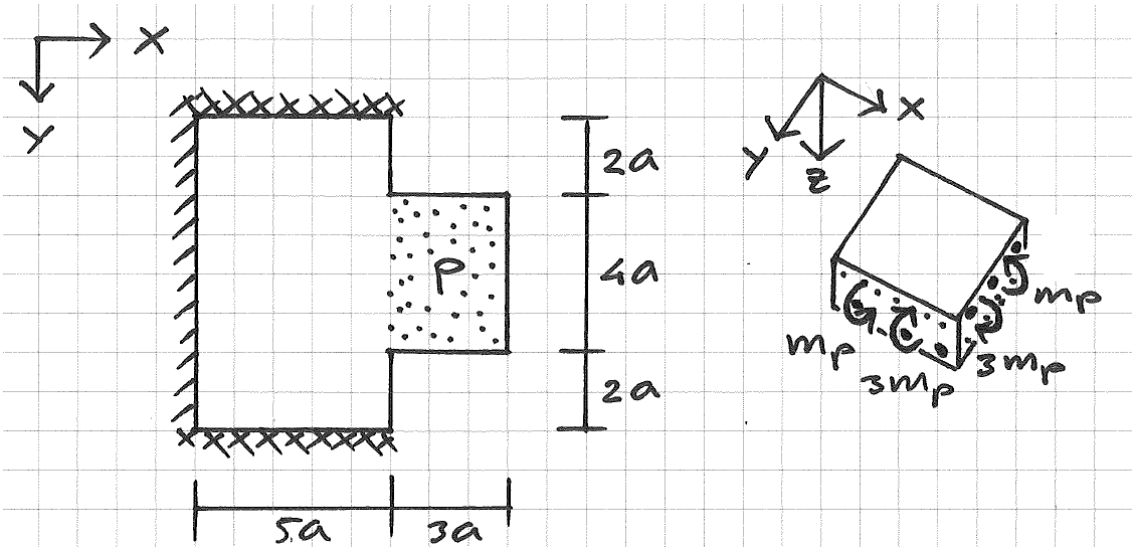

Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

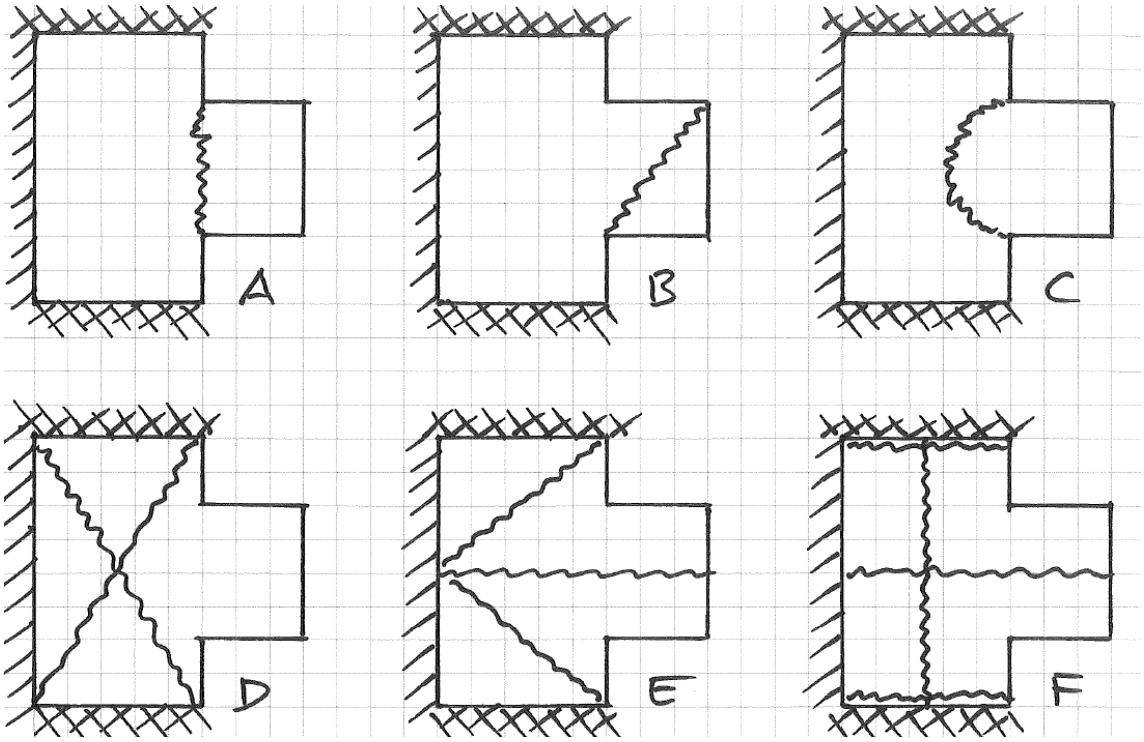

Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an upper bound for p expressed in m_p and a (1.5 point).

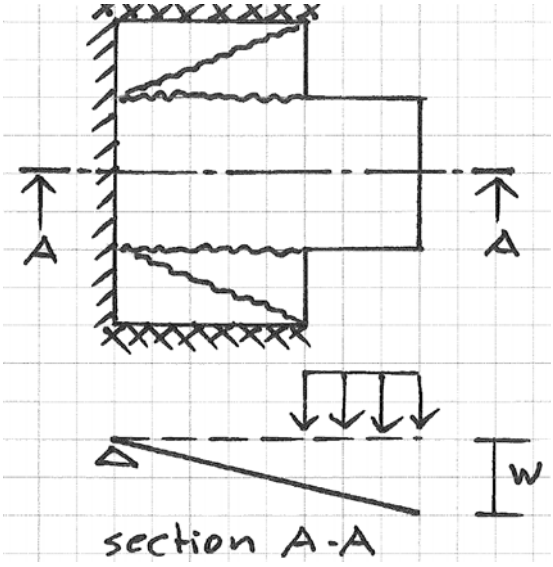

Figure 5. Yield line pattern of problem 2b

c Determine the largest lower-bound for p using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Can shear force (transverse force) be neglected in calculating the moment capacity of most steel beams?
Can shear force be neglected in calculating the moment capacity of most concrete beams?
Choose A, B, C or D (0.5 point).

A – Yes, Yes
B – Yes, No
C – No, Yes
D – No, No

b Do rolling stresses reduce stiffness? Do rolling stresses reduce strength?
Choose A, B, C or D (0.5 point).

A – Yes, Yes
B – Yes, No
C – No, Yes
D – No, No

c For which problems is the exact plastic solution available?
Choose from A, B, C and D (0.5 point).

A – Simply supported square isotropic plate with evenly distributed load
B – Rectangular restrained isotropic plate with evenly distributed load
C – Circular restrained isotropic plate with a point load in the centre
D – Isotropic plate corner with a point load

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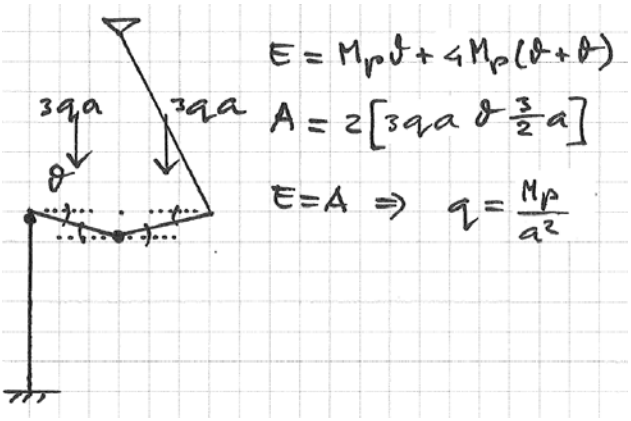
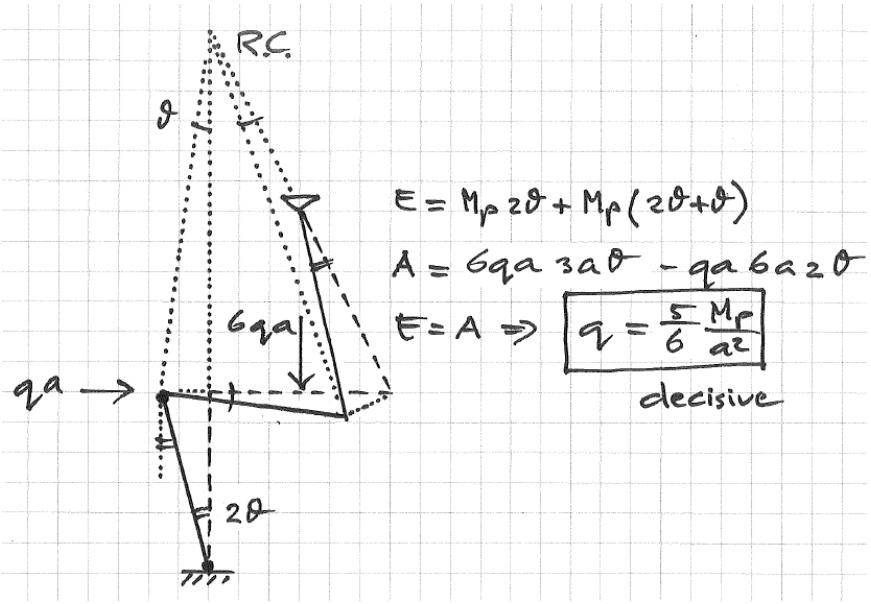
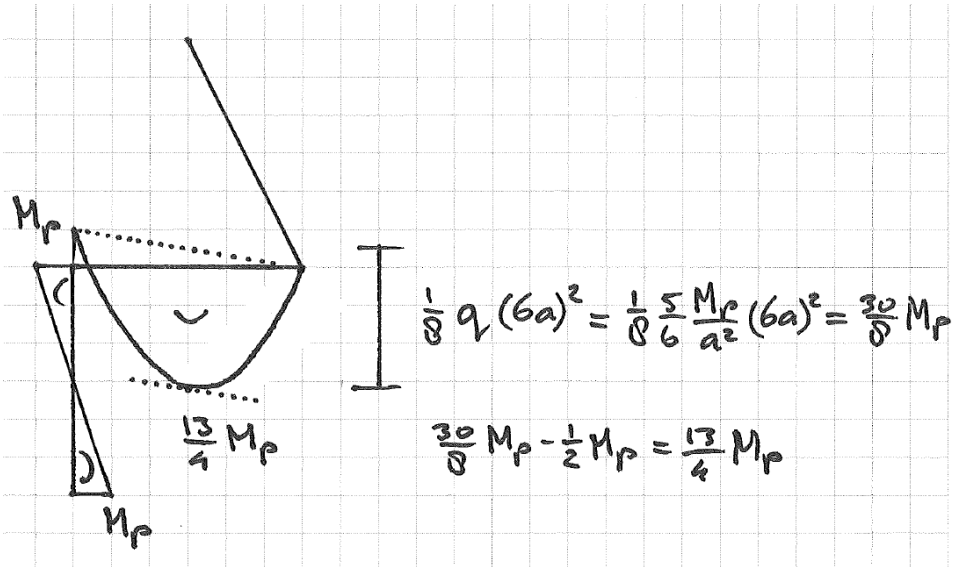
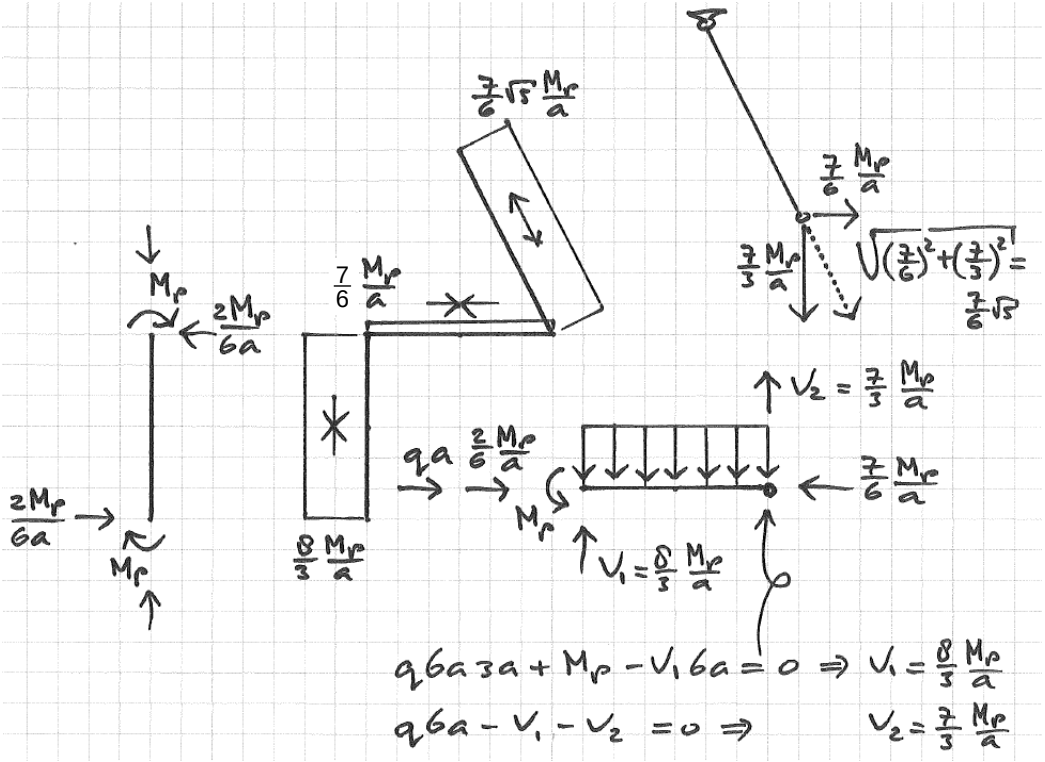
Answer to Problem 1a



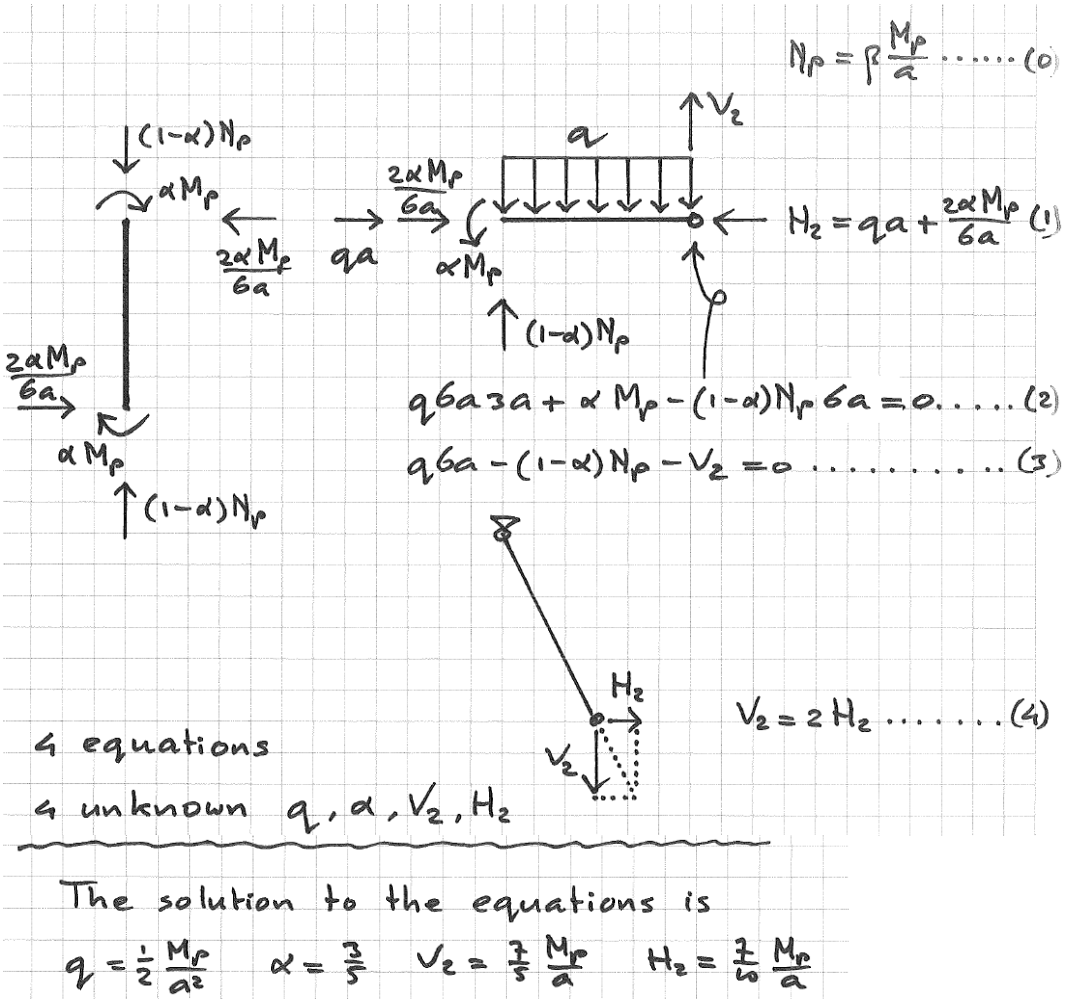


Answer to Problem 1b



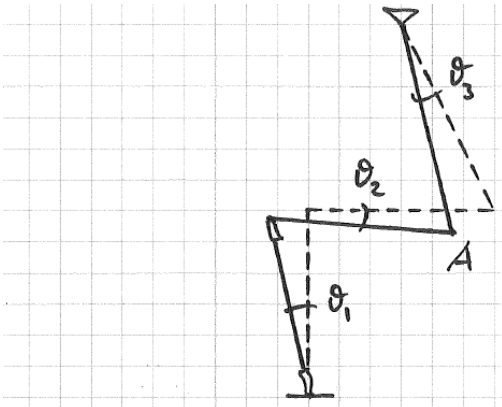


Answer to Problem 1c



The solution to the equations is

$q=\frac{1}{2}\frac{M_p}{a^2}$ $\alpha=\frac{3}{5}$ $V_2=\frac{7}{5}\frac{M_p}{a}$ $H_2=\frac{7}{10}\frac{M_p}{a}$



$\beta = 4$

$u = \vartheta \frac{a}{\beta}$

horizontal displacement of A $\leftarrow$

$\vartheta_1 6a = \vartheta_3 6a$

vertical displacement of A $\downarrow$

$\vartheta_1 \frac{a}{\beta} + (\vartheta_1 + \vartheta_2)\frac{a}{\beta} + \vartheta_2 6a = \vartheta_3 3a$

$E = M_p \vartheta_1 + M_p (\vartheta_1 + \vartheta_2)$

$A = q 6a \left[\vartheta_1 \frac{a}{\beta} + (\vartheta_1 + \vartheta_2)\frac{a}{\beta} + \vartheta_2 3a\right] - qa\, \vartheta_1 6a$

$E = A$

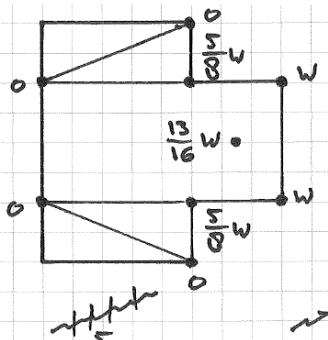
The solution to the equations is

$q = \frac{1}{2}\frac{M_p}{a^2} \quad \vartheta_2 = \frac{2}{5}\vartheta_1 \quad \vartheta_3 = \vartheta_1$

Answer to Problem 2a

A, B, E

Answer to Problem 2b



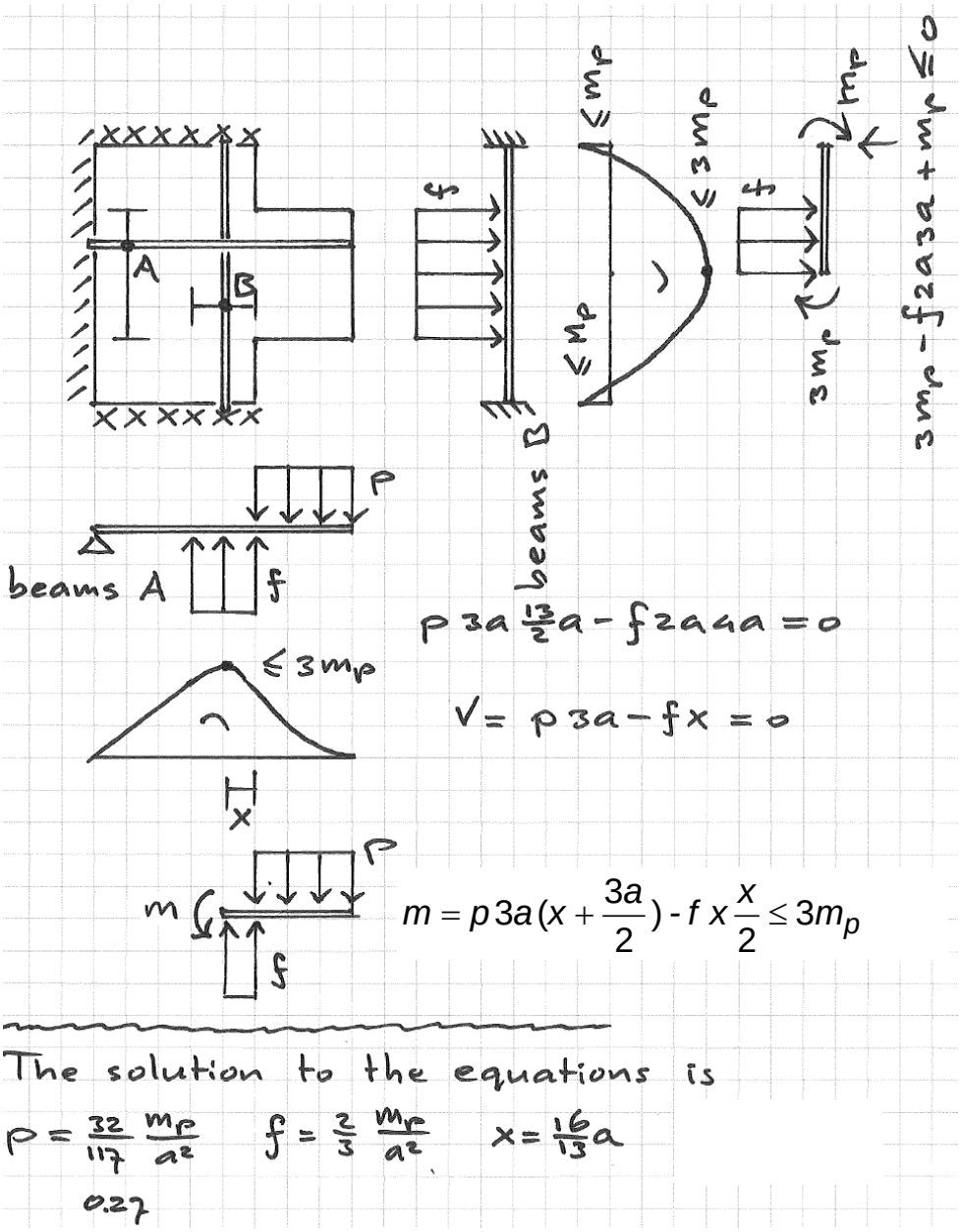
$E = 2\left[m_p \frac{\frac{5}{8}w}{2a} 5a + 3m_p \frac{\frac{5}{8}w}{5a} 2a + 3m_p \frac{\frac{5}{8}w}{2a} 5a\right] = 14 m_p w$

$A = p\, 3a\, 4a \frac{13}{16} w = \frac{39}{4} p a^2 w$

$E = A \Rightarrow p = \frac{56}{39}\frac{m_p}{a^2}$

1.44

Answer to Problem 2c



The solution to the equations is

$p = \frac{32}{117}\frac{m_p}{a^2}$ $f = \frac{2}{3}\frac{m_p}{a^2}$ $x = \frac{16}{13}a$

0.27

Answer to Problem 3a
B

Answer to Problem 3b
B

Answer to Problem 3c
A, C, D