Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 22 January 2015, 14:00 – 17:00 hours

Write your name and study number at the top right-hand of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

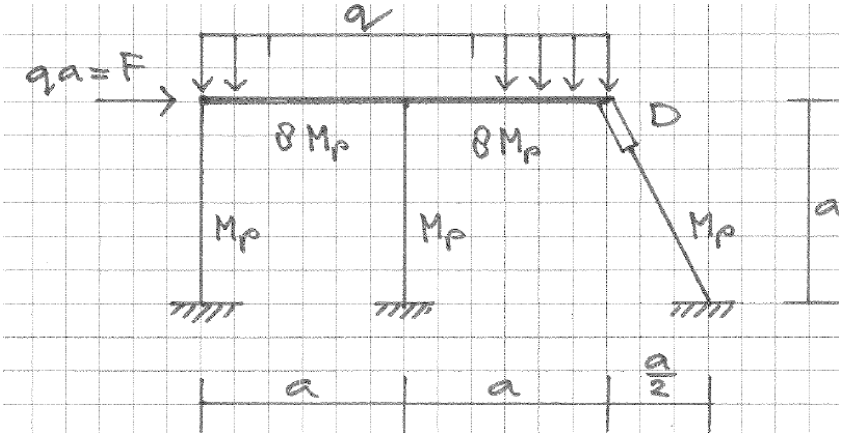

Figure 1. Frame structure

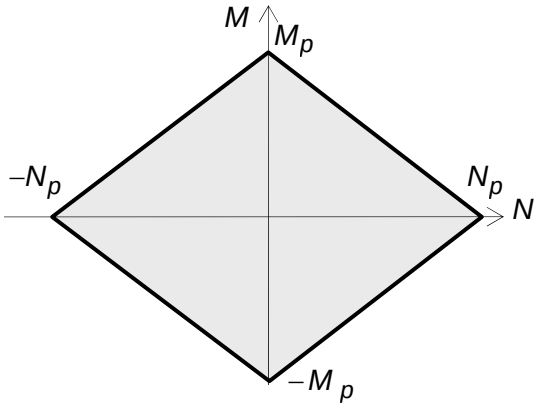

Figure 2. Yield contour

Problem 1

An offshore frame consists of three legs and one beam (Fig.1) The legs have strengths M_p and the beam has a strength $8M_p$. All joints are rigid. The structure is loaded by an inclined evenly distributed load q and a horizontal load F.

The relation of Figure 2 exists between the plastic moments and the plastic normal forces of joint D.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = \frac{4}{\sqrt{5}}$. This relation is for joint D only. There is no interaction in the other members and joints. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
Use the mechanism shown in Figure 3. You only need to write down the equations and not solve the equations (1.5 points).

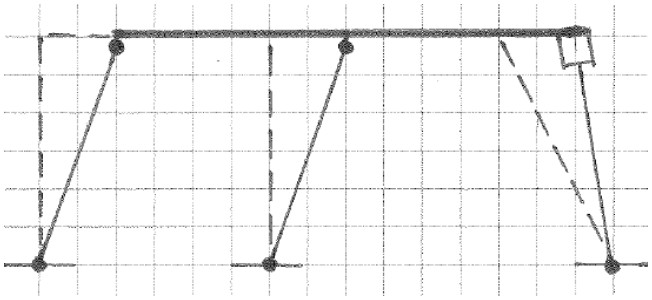

Figure 3. Mechanism for interaction moment and normal force

Problem 2

A reinforced concrete plate has a simply supported edge and a fixed edge (Fig. 4). In the middle it has a large opening. It carries an evenly distributed load *q*. The plate is homogeneous and orthotropic.

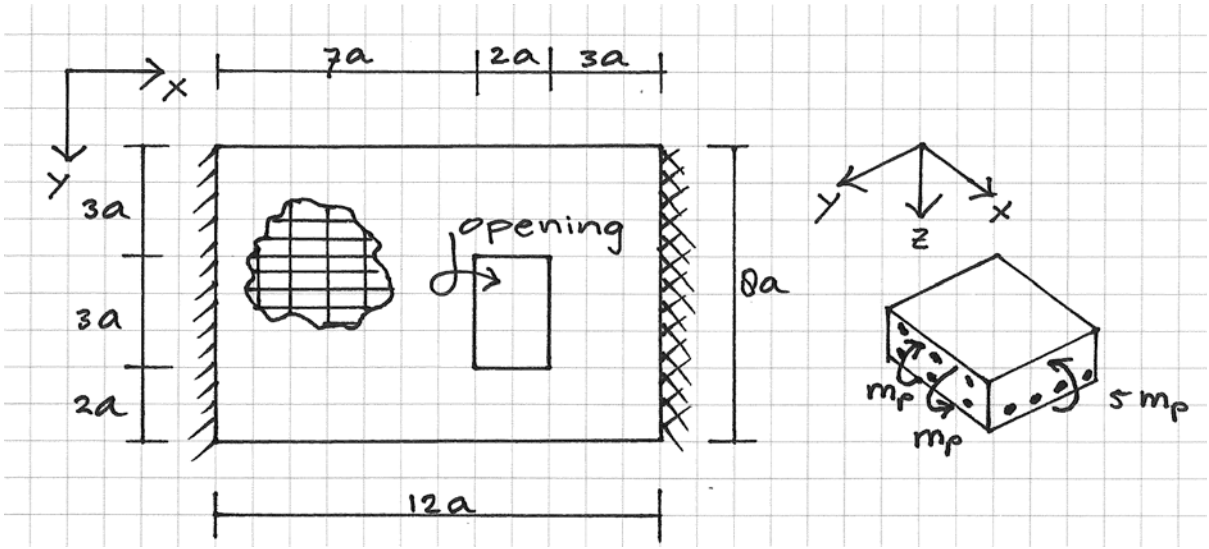

Figure 4. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

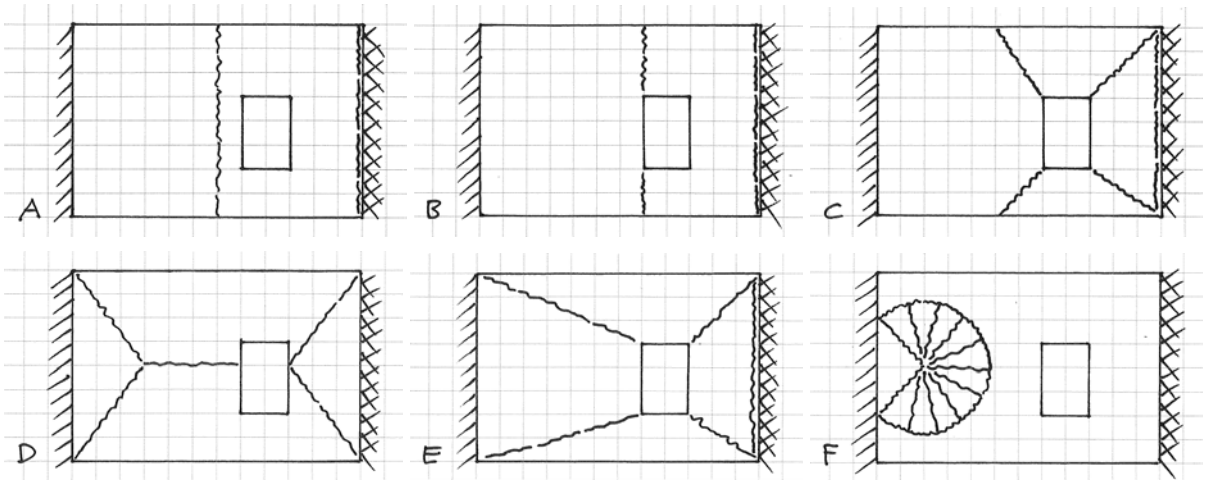

Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 6. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

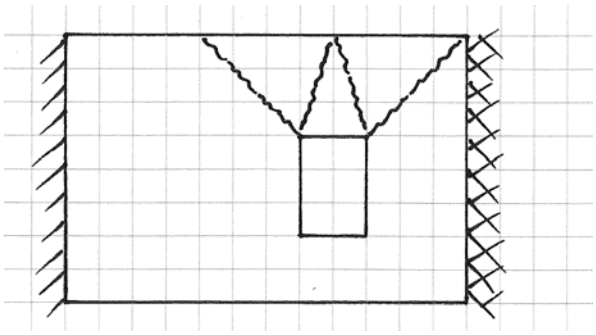

Figure 6. Yield line pattern of problem 2b

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Foundation settlements can be ignored when we calculate the plastic collapse load of a structure. How can this be explained correctly? Choose A, B, C or D (0.5 point)

A – Settlement stresses are very small compared to the stresses due to self-weight.
B – The plastic hinges in the ultimate limit state make the structure statically determinate.
C – When all foundation piles settle the same amount there will be no redistribution of the force flow.
D – Foundation settlements cannot be ignored. It can cause damage which can become worse due to rain and wind. Expensive repairs can be required, the structure might be closed temporarily reducing its serviceability and in time the structure might even collapse.

b Consider the three plates of Figure 7. Each of them is homogeneous and isotropic with a plastic moment m_p. Each plate is loaded by a perpendicular point load F at the location of the dot. Which of the plates has the largest ultimate load? (1 point)

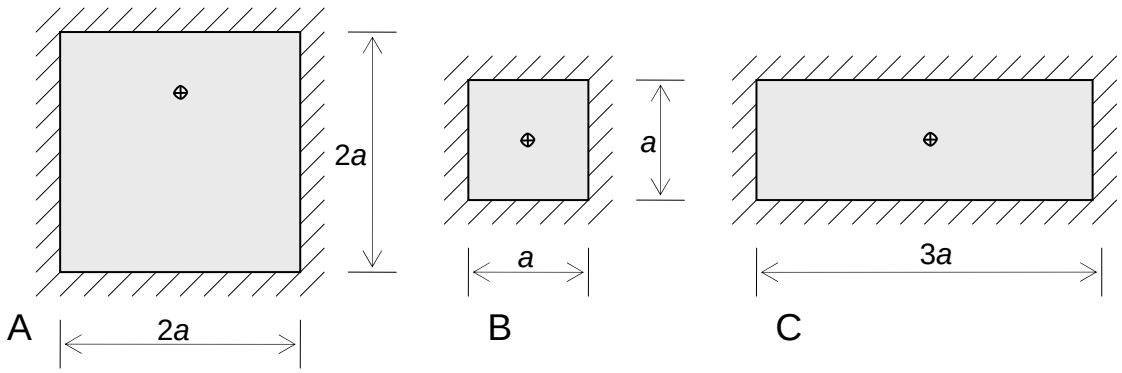

Figure 7. Plates loaded by a point load

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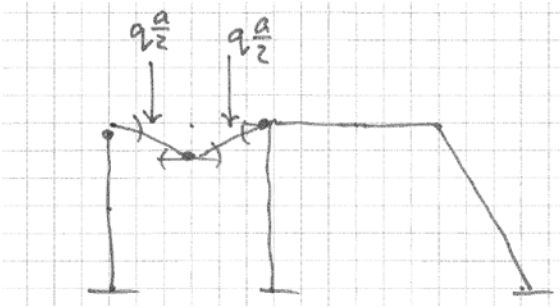
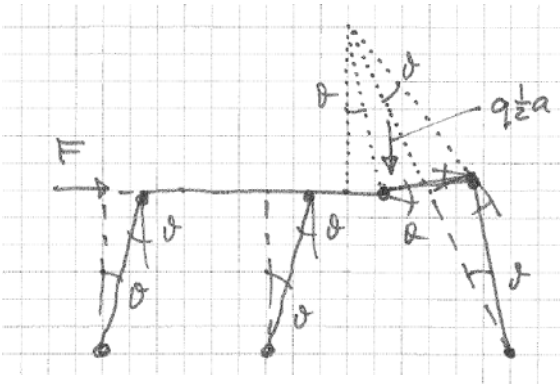
Answers to Problem 1a



$A = 2\left[q\frac{a}{2}\vartheta\frac{a}{4}\right] = \frac{1}{4}qa^2\vartheta$

$E = M_p\vartheta + \vartheta M_p(\vartheta+\vartheta) + \vartheta M_p\vartheta = 25M_p\vartheta$

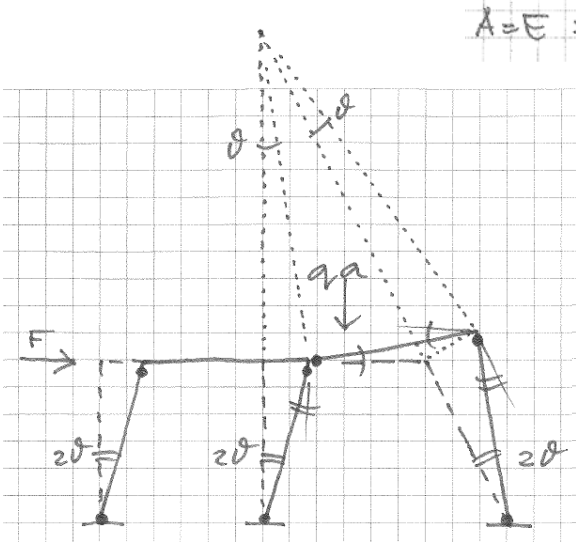
$E = A \Rightarrow \quad q = 100\,\frac{M_p}{a^2}$



$A = Fa\vartheta - q\frac{1}{2}a\,\vartheta\,\frac{1}{4}a = \frac{7}{8}qa^2\vartheta$

$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta + \vartheta M_p\vartheta + M_p(\vartheta+\vartheta) + M_p\vartheta = 15M_p\vartheta$

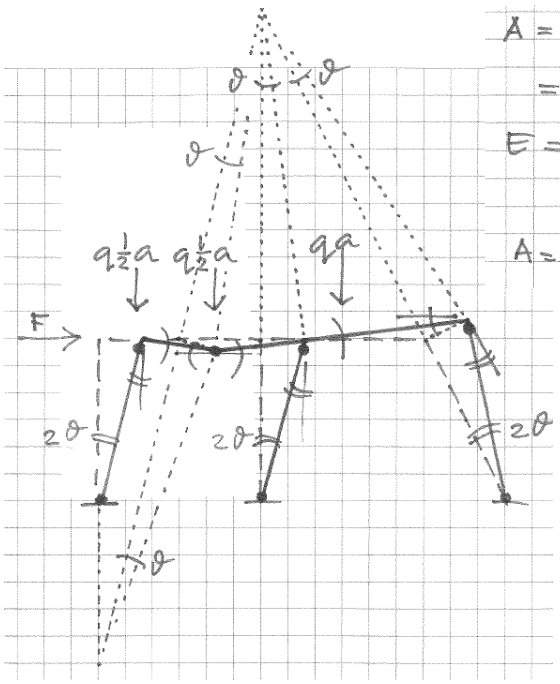
$A = E \Rightarrow \quad q = \frac{120}{7}qa^2$



$A = F\,2\vartheta a - qa\,\vartheta\frac{1}{2}a = \frac{3}{2}qa^2\vartheta$

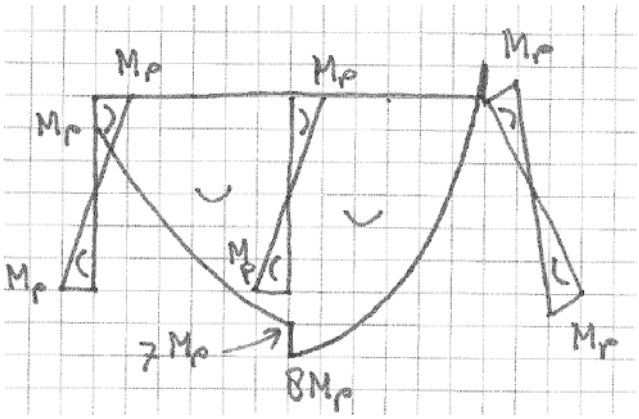
$E = M_p 2\vartheta + M_p 2\vartheta + M_p 2\vartheta + M_p 2\vartheta + \vartheta M_p\vartheta$
$+ M_p(\vartheta+2\vartheta) + M_p 2\vartheta = 21M_p\vartheta$

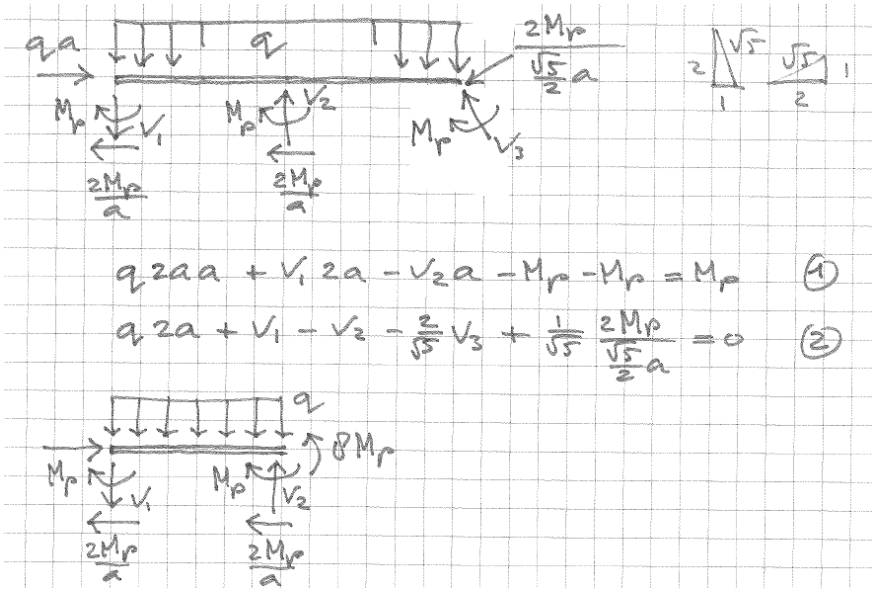
$A = E \Rightarrow \quad \boxed{q = 14\,\frac{M_p}{a^2}}$ decisive



$$A = F\,2\vartheta a + q\tfrac{1}{2}a\,\vartheta\tfrac{1}{4}a + q\tfrac{1}{2}a\,\vartheta\tfrac{1}{4}a - q\,a\,\vartheta\tfrac{1}{2}a$$
$$= \tfrac{7}{4}qa^2\vartheta$$
$$E = M_p 2\vartheta + M_p(2\vartheta-\vartheta) + 8M_p(\vartheta+\vartheta) + M_p 2\vartheta +$$
$$+ M_p(2\vartheta+\vartheta) + M_p(2\vartheta+\vartheta) + M_p 2\vartheta = 29 M_p\vartheta$$
$$A = E \Rightarrow \quad q = \frac{116}{7}\frac{M_p}{a^2}$$

Answer to Problem 1b



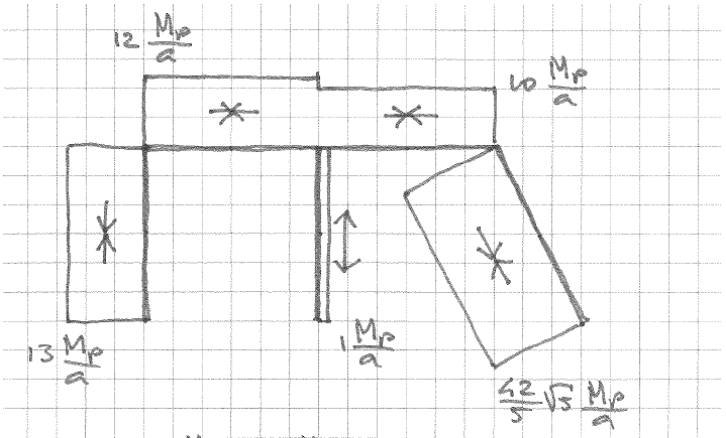


$$q\,2a\,a + V_1\,2a - V_2 a - M_p - M_p = M_p \qquad (1)$$
$$q\,2a + V_1 - V_2 - \frac{2}{\sqrt{5}}V_3 + \frac{1}{\sqrt{5}}\frac{2M_p}{\frac{\sqrt{5}}{2}a} = 0 \qquad (2)$$

$$qa\,a\tfrac{1}{2} + V_1 a - M_p - M_p + 8M_p = 0 \qquad (3)$$

The solution to equation (1), (2) and (3) is

$V_1 = -13\frac{M_p}{a}$ $\quad V_2 = -\frac{M_p}{a}$ $\quad V_3 = \frac{42}{5}\sqrt{5}\frac{M_p}{a}$



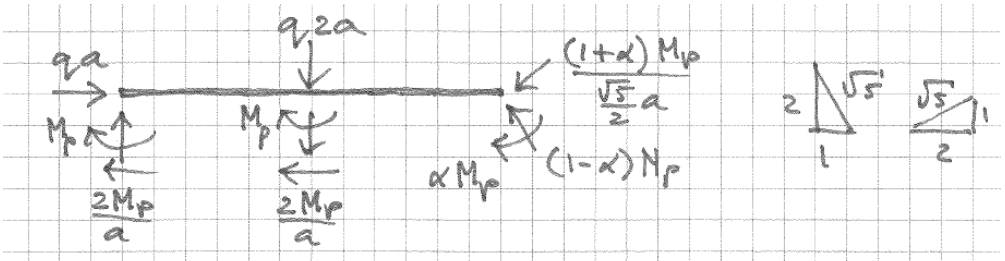
$$qa - \frac{2M_p}{a} = 12\frac{M_p}{a}$$

$$qa - \frac{2M_p}{a} - \frac{2M_p}{a} = 10\frac{M_p}{a}$$

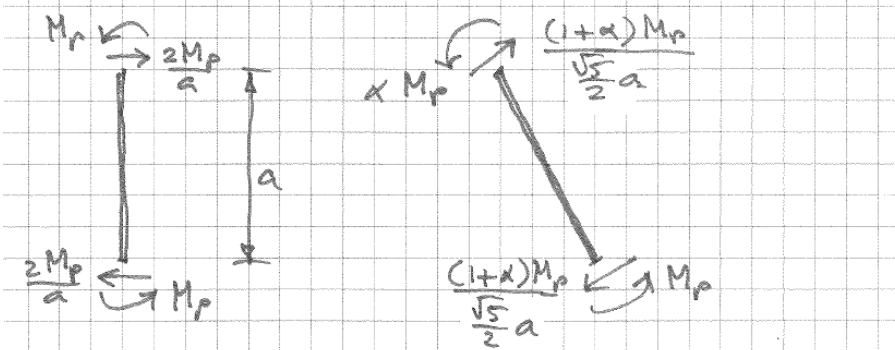
check:

$$\frac{2}{\sqrt{5}}\frac{2M_p}{\frac{\sqrt{5}}{2}a} + \frac{1}{\sqrt{5}}\frac{42}{2}\sqrt{5}\frac{M_p}{a} = 10\frac{M_p}{a} \quad \text{OK.}$$

Answer to Problem 1c, Lower bound



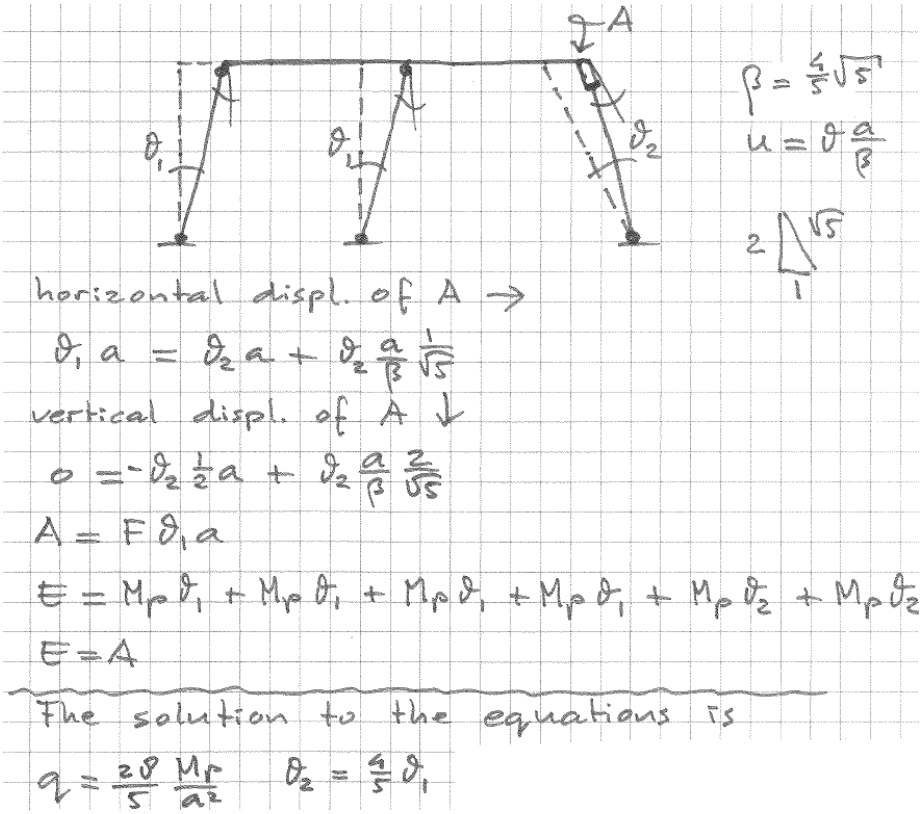
$$qa = \frac{2M_p}{a} + \frac{2M_p}{a} + \frac{1}{\sqrt{5}}(1-\alpha)N_p + \frac{2}{\sqrt{5}}\frac{(1+\alpha)M_p}{\frac{\sqrt{5}}{2}a} \qquad (1)$$



The solution to the equation is

$q = \frac{28}{5}\frac{M_p}{a^2}$. ($\alpha$ drops out.)

Answer to Problem 1c, Upper bound



Problem 1 represents an offshore rig that needs to survive earthquakes. Joint D is a crumble zone. It can carry service loading elastically. In an earthquake it deforms plastically to absorb energy. It also prevents that the horizontal beam deforms plastically. After an earthquake it can be replaced.

Answers to Problem 2

See exam 20 January 2011

Answers to Problem 3

See exam 20 June 2007