Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 7 April 2016, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

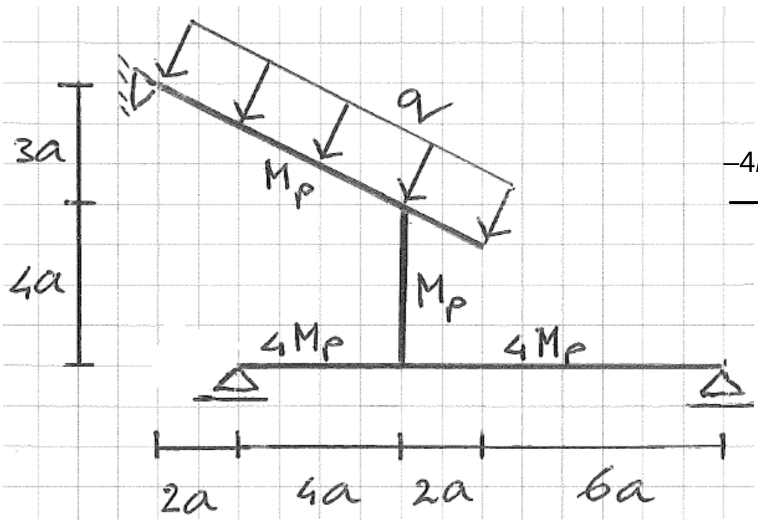


Figure 1. Frame structure

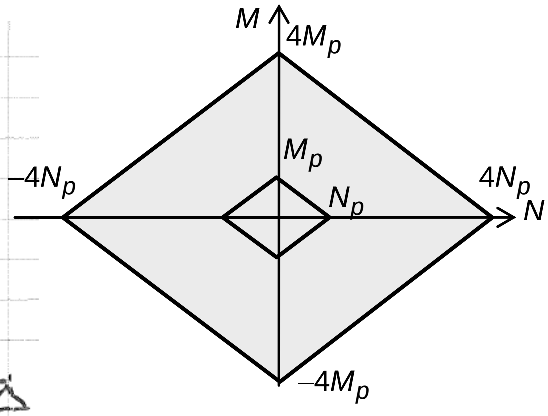


Figure 2. Yield contours

Problem 1

A frame consists of two beams and one column (Fig.1) The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 6. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has simply supported edges (Fig. 3). It carries an evenly distributed load *p.* The top and bottom reinforcement have a strength m_p and the bars are directed in parallel to the edges. The plate is homogeneous and isotropic.

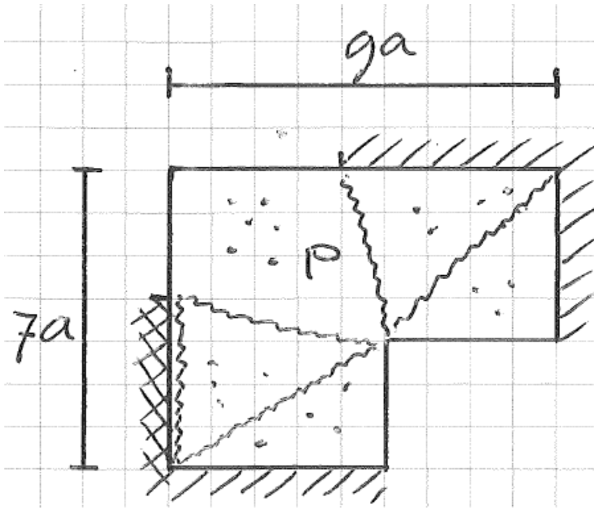


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

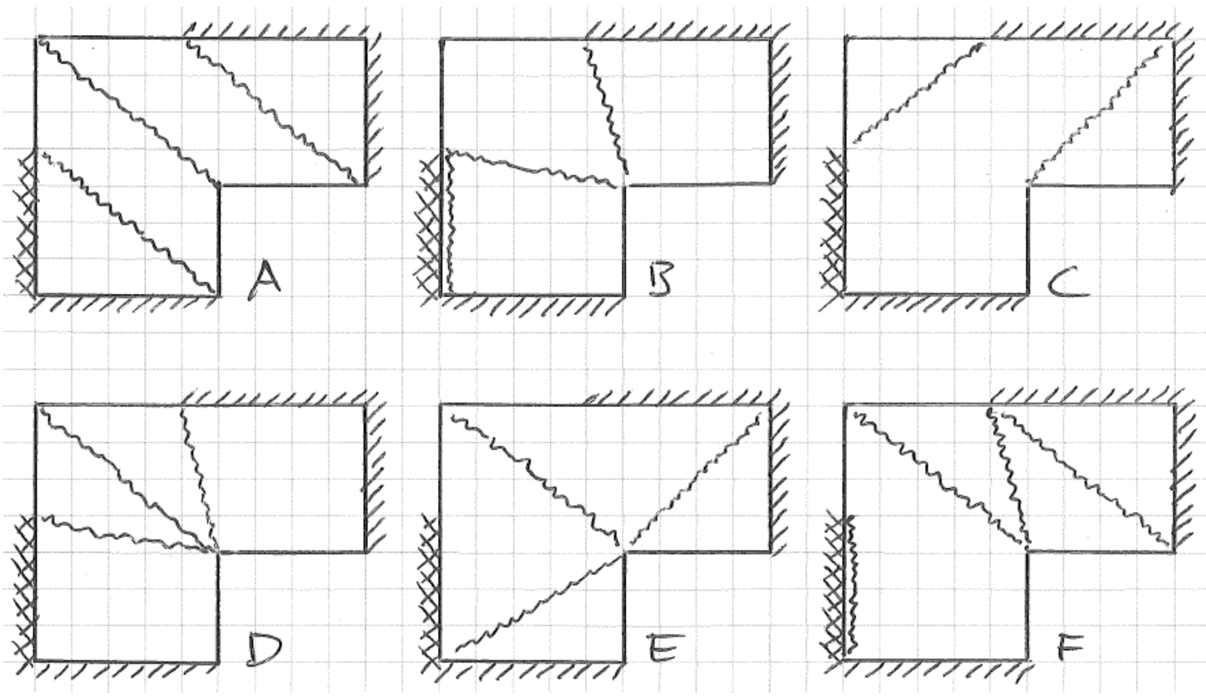


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for p expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for p using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Consider a plate that is fixed at all edges. The plate has a strength m_p in all directions. The plate is loaded by a point load perpendicular to the plate. What is the collapse load? Choose A, B, C or D (0.5 point).

A $2\pi m_p$

B $(2+\pi) m_p$

C $4\pi m_p$

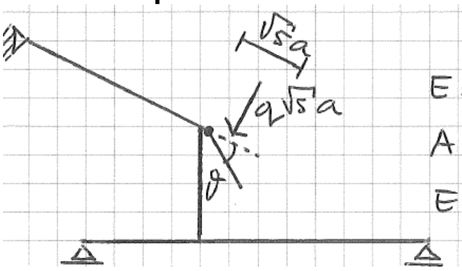
D $6 m_p$

b The Mohr-Coulomb yield contour is suitable for which material?
Choose from A, B, C and D (0.5 point).

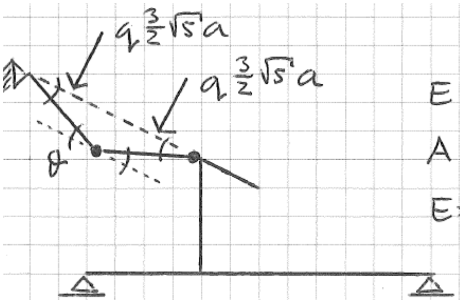
A Reinforced concrete
B Unreinforced concrete
C Coper
D Kevlar

c Codes of practice do not require that foundation settlements are considered in structural analysis for the ultimate limit state. Many engineers are worried about this because foundation settlements occur often and can give very large linear elastic stresses. Do we need to worry about ignoring foundation settlements? Explain your answer (0.5 point).

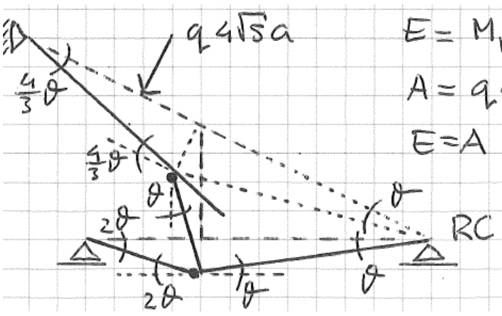
Answer to problem 1a



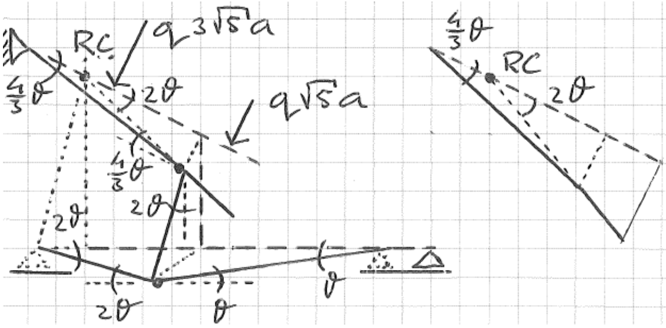
$$E = M_p\,\theta$$
$$A = q\sqrt{5}a\,\theta\,\tfrac{1}{2}\sqrt{5}a$$
$$E = A \;\Rightarrow\; q = \tfrac{2}{5}\frac{M_p}{a^2}$$



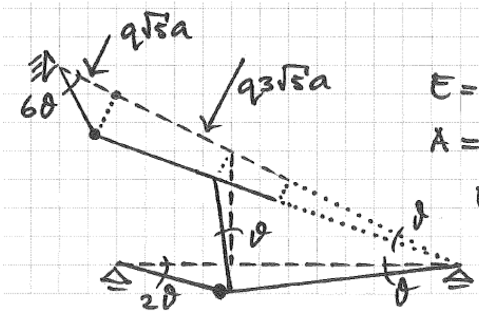
$$E = M_p(\theta+\theta) + M_p\,\theta$$
$$A = 2\left(q\tfrac{3}{2}\sqrt{5}a\,\theta\,\tfrac{1}{2}\tfrac{3}{2}\sqrt{5}a\right)$$
$$E = A \;\Rightarrow\; q = \tfrac{4}{15}\frac{M_p}{a^2}$$



$$E = M_p\left(\tfrac{4}{3}\theta+\theta\right) + 4M_p(2\theta+\theta)$$
$$A = q4\sqrt{5}a\,\tfrac{4}{3}\theta\,2\sqrt{5}a$$
$$E = A \;\Rightarrow\; q = \tfrac{43}{160}\frac{M_p}{a^2}$$

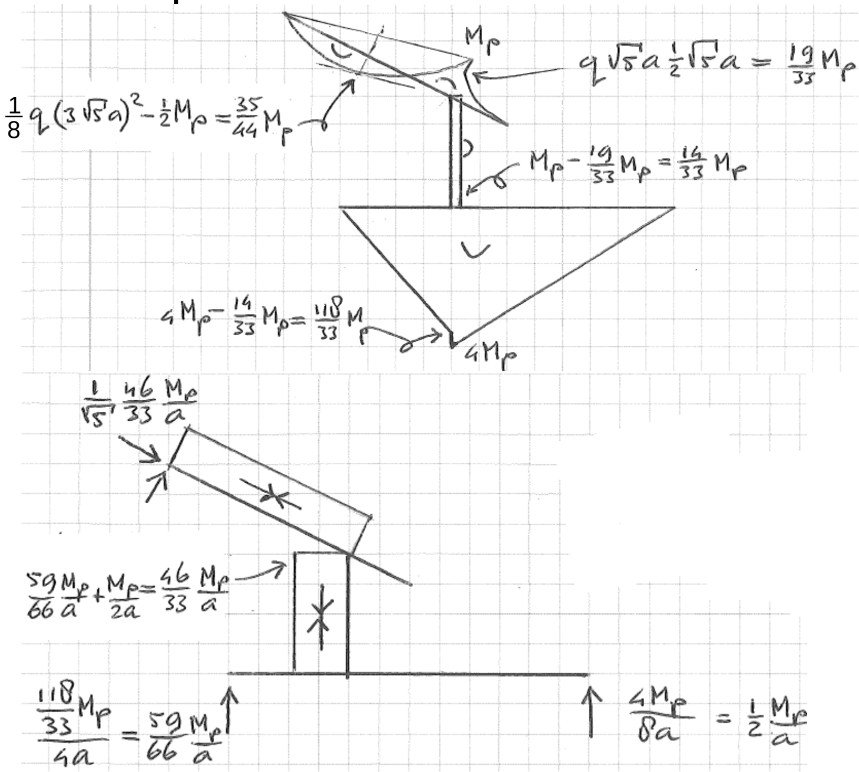


$$E = 4M_p(2\theta+\theta) + M_p\left(2\theta-\tfrac{4}{3}\theta\right)$$
$$A = q3\sqrt{5}a\,\tfrac{4}{3}\theta\,\tfrac{1}{2}3\sqrt{5}a + q\sqrt{5}a\,2\theta\,\tfrac{5}{2}\sqrt{5}a$$
$$E = A \;\Rightarrow\; \boxed{q = \tfrac{38}{165}\frac{M_p}{a^2}}$$ decisive

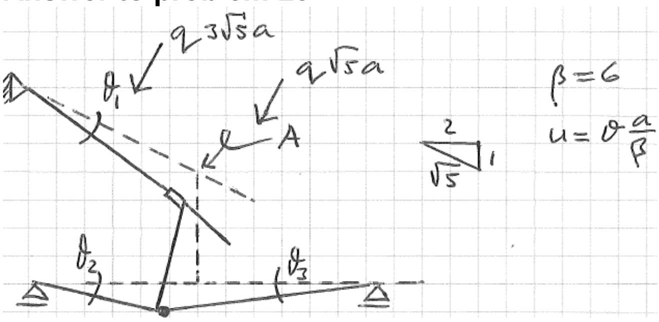


$$E = M_p(6\theta+\theta) + 4M_p(2\theta+\theta)$$
$$A = q\sqrt{5}a\,6\theta\,\tfrac{1}{2}\sqrt{5}a + q3\sqrt{5}a\,\theta\,\tfrac{9}{2}\sqrt{5}a$$
$$E = A \;\Rightarrow\; q = \tfrac{38}{165}\frac{M_p}{a^2}$$
also decisive

Answer to problem 1b



Answer to problem 1c



$\vartheta_2 = 2\vartheta_3$

vert. displ. of A ↓

$\vartheta_1 6a - (\vartheta_2 - \vartheta_1)\frac{a}{\beta}\frac{1}{\sqrt{5}} = \vartheta_2 4a$

$E = M_p(\vartheta_2 - \vartheta_1) + 4M_p(\vartheta_2 + \vartheta_3)$

$A = q3\sqrt{5}a\,\vartheta_1\frac{1}{2}3\sqrt{5}a + q\sqrt{5}a\left(\vartheta_1 3\sqrt{5}a + \vartheta_2\frac{1}{2}\sqrt{5}a\right)$

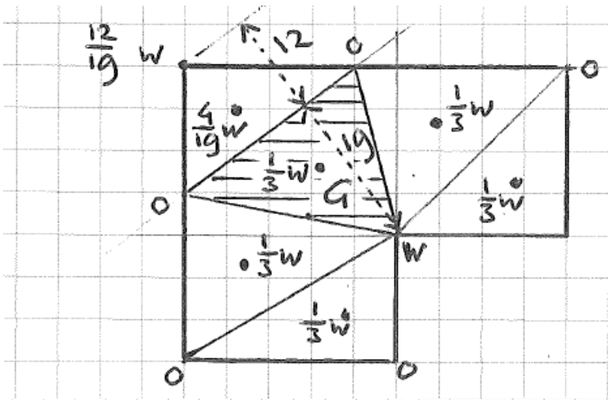
$E = A$

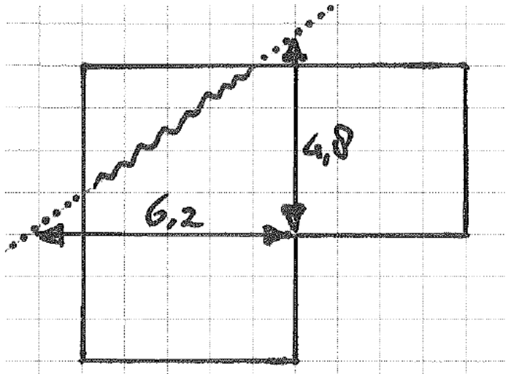
The solution to the equations is $q = 0.229\frac{M_p}{a^2}$

Answer to problem 2a

A C D

Answer to problem 2b





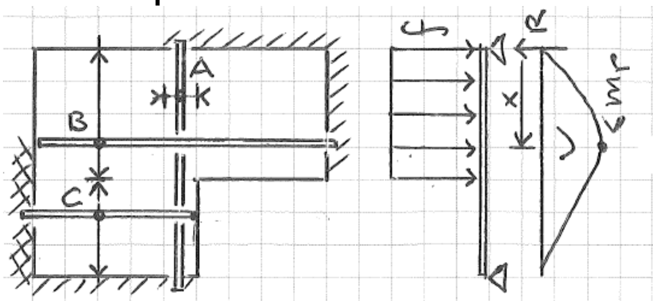
$$A = p\tfrac{1}{2}5a\,3a\,\tfrac{1}{3}w + p\tfrac{1}{2}4a\,5a\,\tfrac{1}{3}w - p\tfrac{1}{2}4a\,3a\,\tfrac{4}{19}w + p\tfrac{1}{2}5a\,4a\,\tfrac{1}{3}w + p\tfrac{1}{2}4a\,4a\,\tfrac{1}{3}w + pG\tfrac{1}{3}w$$

$$G = 5a\,4a - \tfrac{1}{2}a\,5a - \tfrac{1}{2}a\,4a - \tfrac{1}{2}3a\,4a$$

$$E = m_p 3a\frac{w}{5a} + m_p 5a\frac{w}{3a} + m_p 4a\frac{w}{4a} + m_p 4a\frac{w}{4a} + m_p 5a\frac{w}{4{,}8a} + m_p a\left(\frac{w}{5a} - \frac{w}{6{,}2a}\right) + m_p 4a\frac{w}{6{,}2a} + m_p a\left(\frac{w}{4a} - \frac{w}{4{,}8a}\right) + m_p 4a\frac{w}{5a}$$

$$E = A \Rightarrow \quad p = 0.497\,\frac{m_p}{a^2}$$

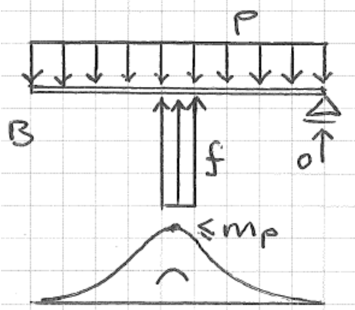
Answer to problem 2c



$$f\,4a\,5a - R\,7a = 0$$

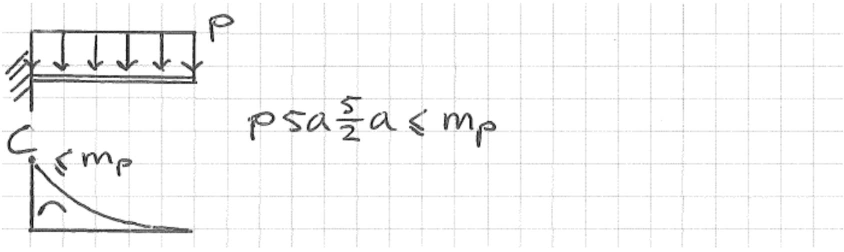
$$f x = R$$

$$R x - f x \frac{x}{2} \le m_p$$



$$p\,9a = f a$$

$$p\frac{9}{2}a\frac{9}{4}a - f\frac{1}{2}a\frac{1}{4}a \le m_p$$



The solution to the equations is $p = \frac{49}{1800}\frac{m_p}{a^2}$

Answer to problem 3

C B

Not for ductile structures. When the structure yields due to another load it becomes statically determined and the settlement stresses disappear.