Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 13 April 2017, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

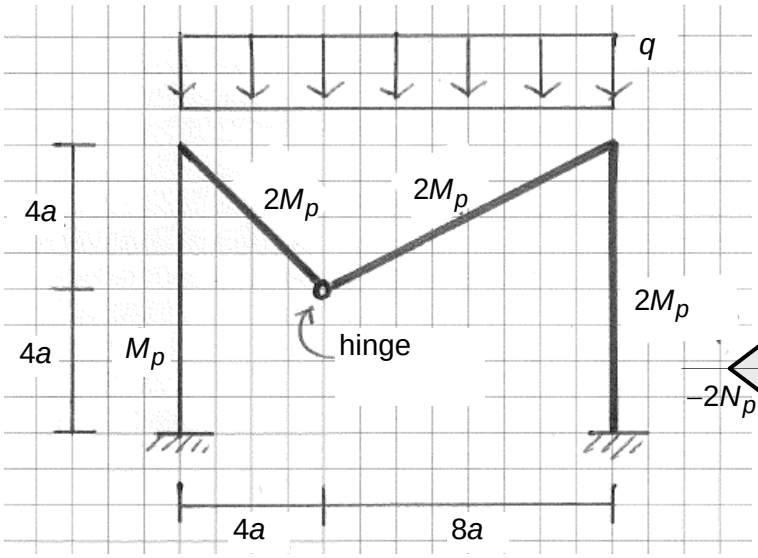


Figure 1. Frame structure

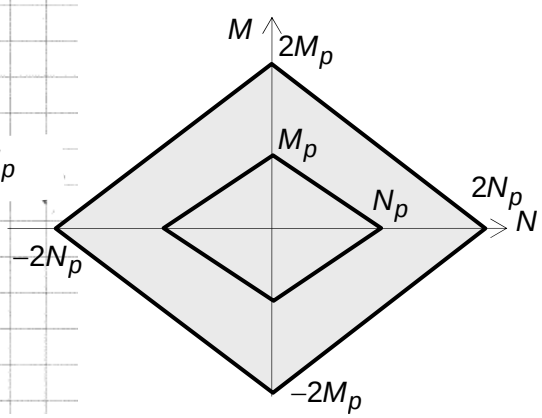


Figure 2. Yield contours

Problem 1

A frame consists of two columns and two beams (Fig.1) All elements have a strength $2M_p$ except for the left column which has a strength M_p. All elements are rigidly connected except for the beams which are connected together by a hinge. The structure is loaded by an evenly distributed line load q (snow). The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 13$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has two simply supported edges (Fig. 3). It carries an evenly distributed line load q [kN/m]. There is no other load on the plate. The plate is homogeneous and orthotropic.

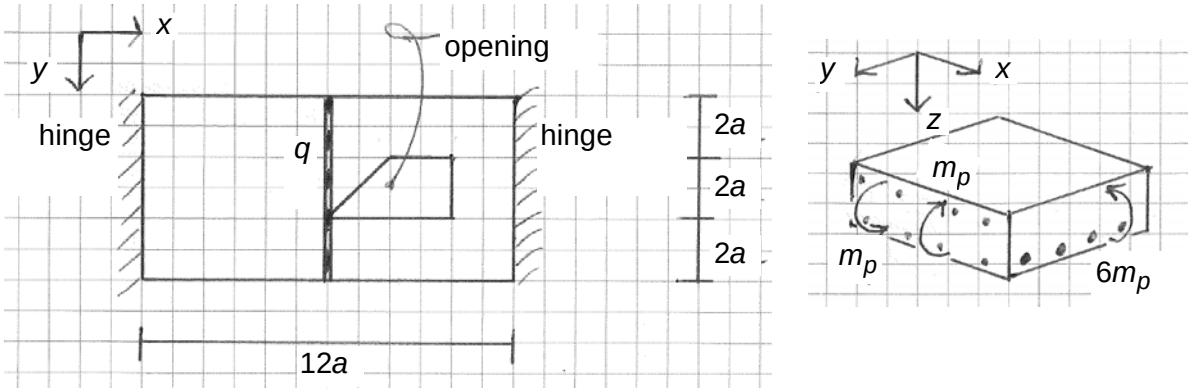


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

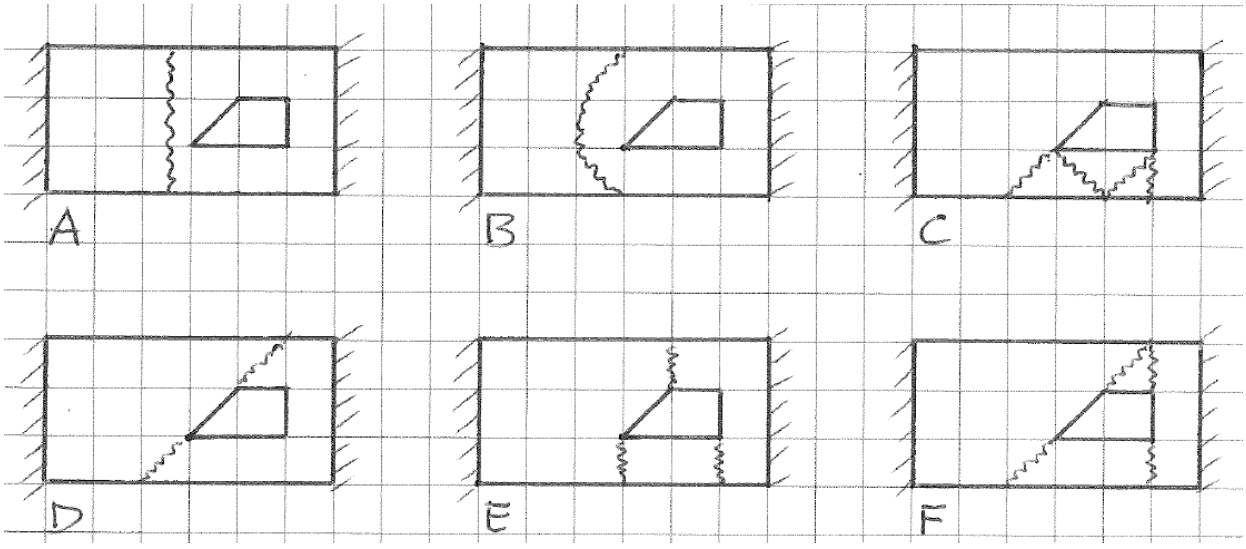


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

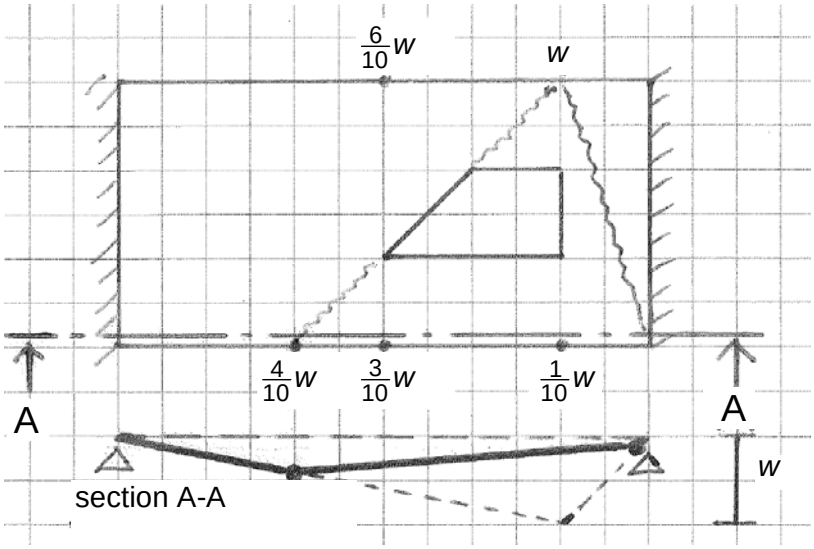


Figure 5. Mechanism of problem 2b

Problem 3

a A straight floor edge is supported by a column. The maximum column force is …
Choose A, B, C or D (0.5 point).

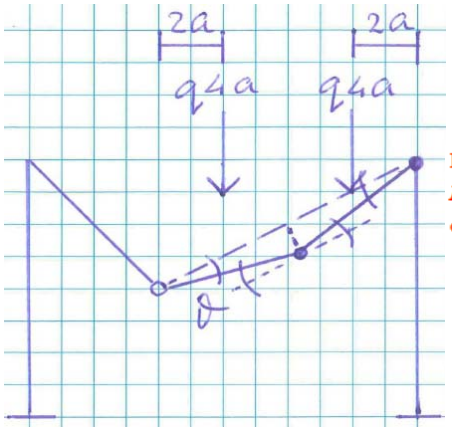
A $2\pi m_p$
B $6m_p$
C $5.6\,m_p$
D $(2+\pi)m_p$

b A square floor is simply supported on all edges. The load is evenly distributed. How much of the collapes load is carried by m_{xy} ? Choose A, B, C or D (0.5 point).

A 0%
B 2π%
C 10%
D 33.3%

c Yield line analysis gives an upperbound of the collapse load of a reinforced concrete floor. However, in reality the collapse load can be much larger. What causes this? (0.5 point)

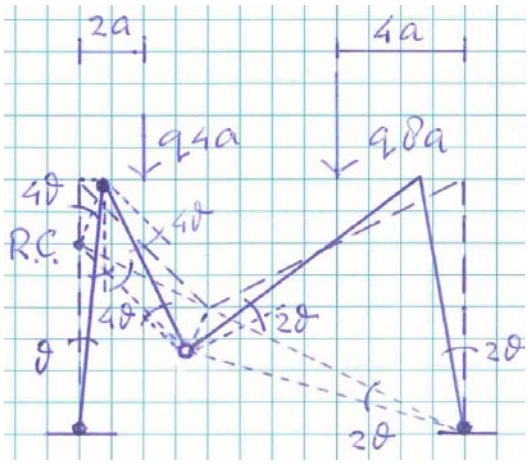
Answers to problem 1a




```
E:= 2*Mp*(t+t)+2*Mp*t:
A:=q*4*a *2*a *t +q*4*a *2*a *t:
q:=solve(E=A,q); evalf(q);
```

$$q := \frac{3\,Mp}{8\,a^2}$$

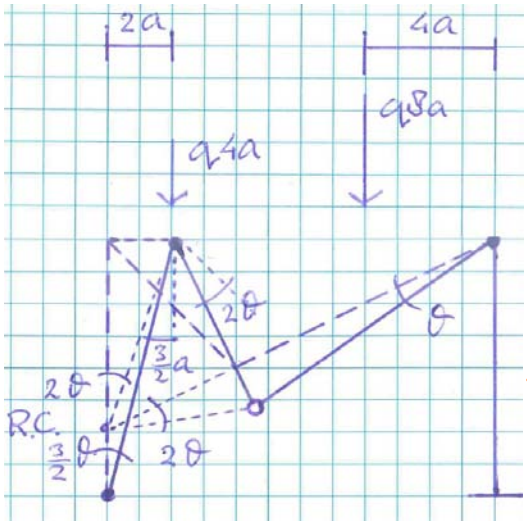
$$\frac{0.3750000000\,Mp}{a^2}$$




```
E:=Mp*t+Mp*(4*t-t)+2*Mp*2*t:
A:=q*4*a *2*a * 4*t +q*8*a *4*a *2*t:
q:=solve(E=A,q); evalf(q);
```

$$\boxed{q := \frac{Mp}{12\,a^2}}$$ decisive

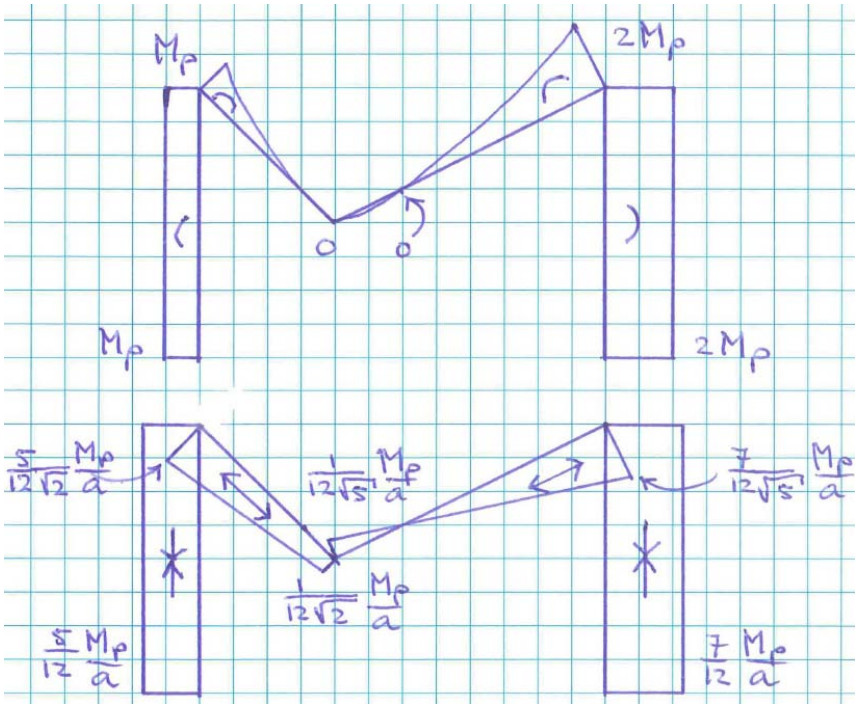
$$\frac{0.08333333333\,Mp}{a^2}$$



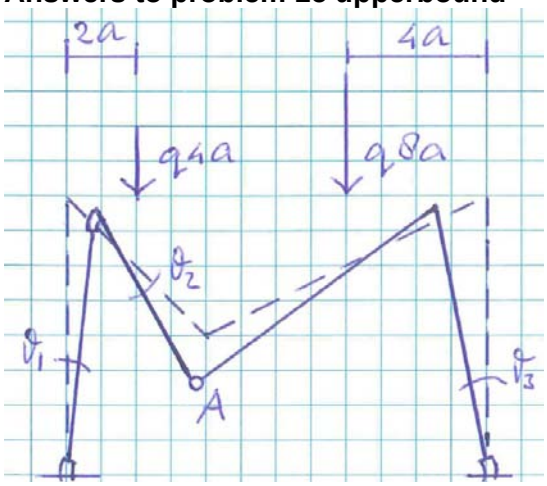

```
E:=Mp*3/2*t+Mp*(2*t-3/2*t)+2*Mp*t:
A:=q*4*a *2*a *2*t +q*8*a *4*a *t:
q:=solve(E=A,q); evalf(q);
```

$$\boxed{q := \frac{Mp}{12\,a^2}}$$ also decisive

Answers to problem 1b



Answers to problem 1c upperbound

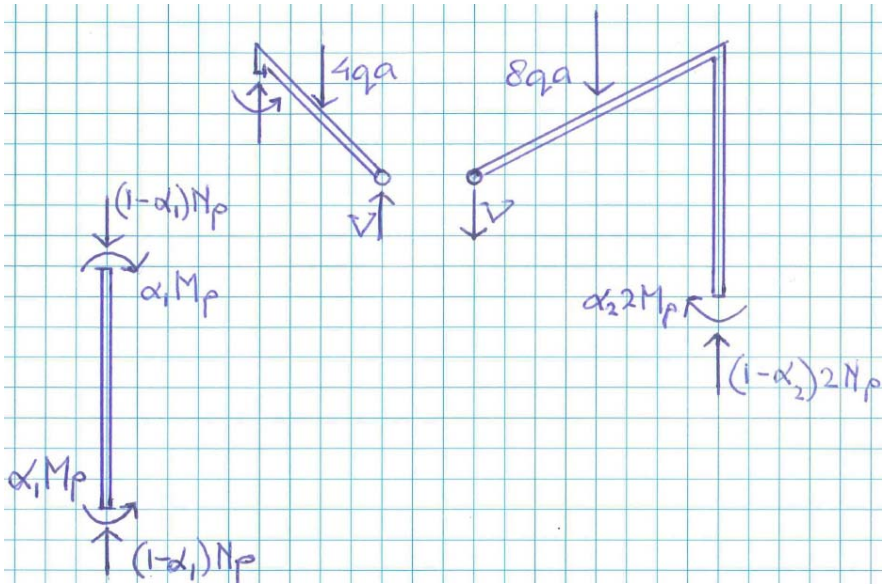



```
b:=13:
# horizontal displacement of A (to the right is positive)
eq1:= t1*8*a -t2*4*a = -t3*4*a:
# vertical displacement of A (down is positive)
eq2:= t1*a/b + (t2-t1)*a/b + t2*4*a = t3*a/b +t3*8*a:
E:= Mp*t1 +Mp*(t2-t1) +2*Mp*t3:
A:= q*4*a *(t1*a/b + (t2-t1)*a/b +t2*2*a) +q*8*a *(t3*a/b +t3*4*a ):
opl:=solve({eq1,eq2,E=A},{q,t2,t3}); assign(opl): q:=evalf(q);
```

$$opl := \{q = \frac{2743\ Mp}{33812\ a^2}, t2 = \frac{105\ t1}{26}, t3 = \frac{53\ t1}{26}\}$$

$$q := \frac{0.08112504436\ Mp}{a^2}$$

Answers to problem 1c lowerbound




```
b:=13:
Np:=b*Mp/a:
eq1:=4*q*a=V+(1-a1)*Np:
eq2:=a1*Mp=4*q*a *2*a-V*4*a:
eq3:=8*q*a=(1-a2)*2*Np-V:
eq4:=a2*2*Mp=8*q*a*4*a+V*8*a:
solve({eq1,eq2,eq3,eq4},{q,a1,a2,V});
```

$$\{V = -\frac{676\,Mp}{8453\,a},\ a1 = \frac{8190}{8453},\ a2 = \frac{8268}{8453},\ q = \frac{2743\,Mp}{33812\,a^2}\}$$

Answers to problem 2a

A C E F

6 right = 1 point
5 right = 0.67 point
4 right = 0.33 point
1, 2 or 3 right = 0 point

Answer to problem 2b

```
E:=6*mp *4*a *( (4/10*w)/(4*a) +(4/10*w)/(8*a)) +6*mp *6*a *( w/(2*a) -(4/10*w)/(8*a) ) +mp *6*a *(w-1/10*w)/(6*a);
```

$$E := \frac{207\,mp\,w}{10}$$

```
A:=q*4*a *6/10*w +q*2*a *9/20*w;
```

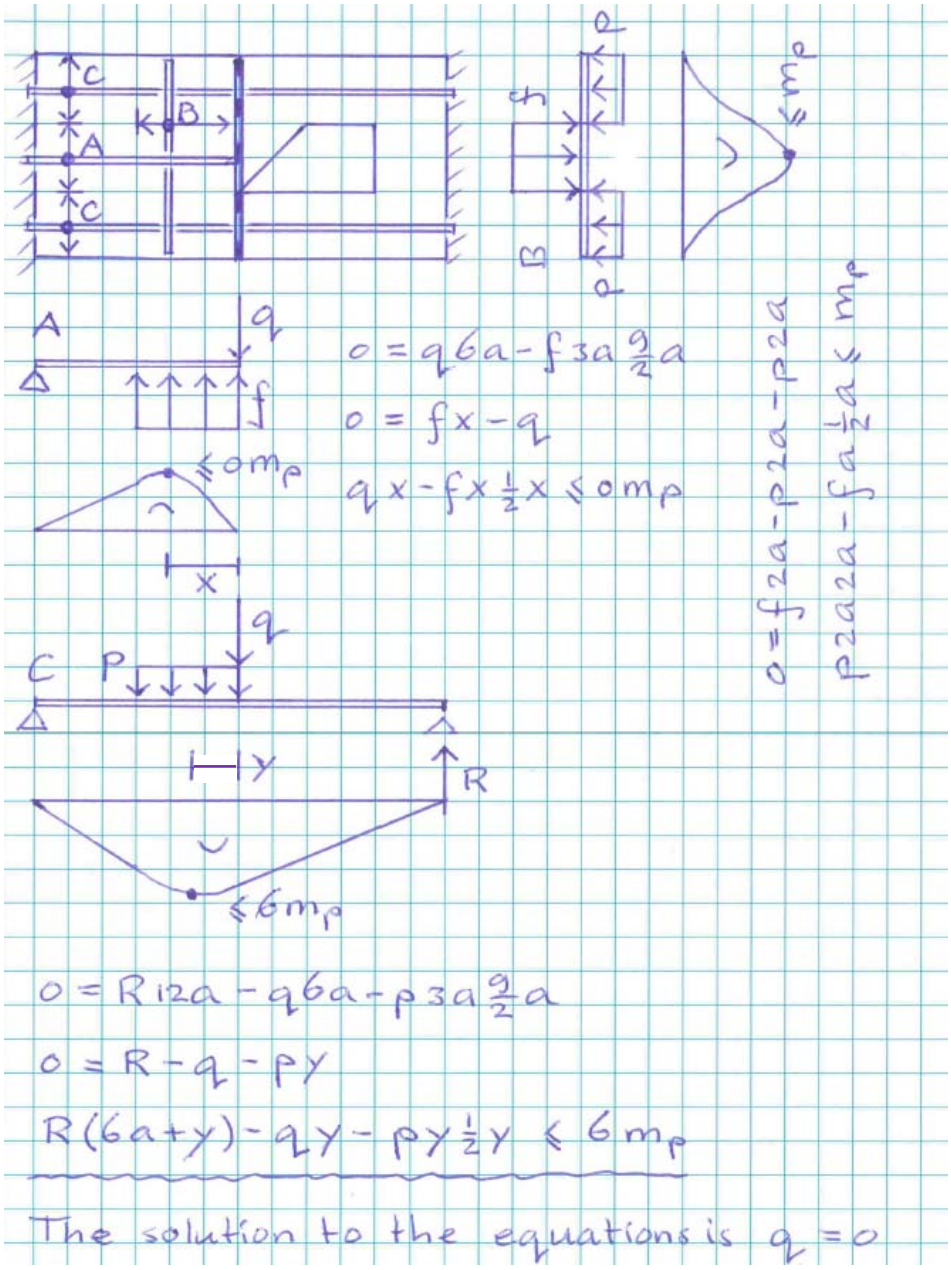
$$A := \frac{33\,q\,a\,w}{10}$$

```
q:=solve(E=A,q); evalf(q);
```

$$q := \frac{69\,mp}{11\,a}$$

$$\frac{6.272727273\,mp}{a}$$

Answer to problem 2c



… because there is no top reinforcement in the *x* direction.

Answers to problem 3

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D …… reader plates page 21
Second order effect / Dome effect …… reader frames page 94