Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 26 January 2017, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

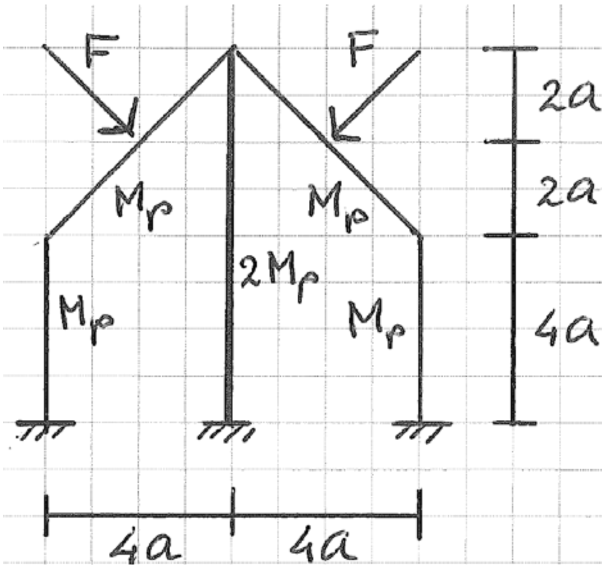


Figure 1. Frame structure

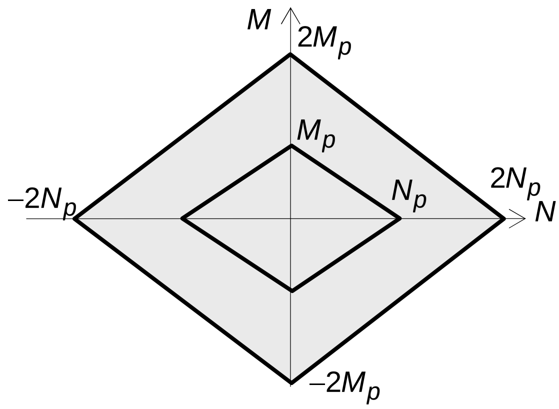


Figure 2. Yield contours

Problem 1

A frame consists of three columns and two beams (Fig.1) All elements have a strength M_p except for the middle column which has a strength $2M_p$. The columns, beams and foundation are rigidly connected. The structure is loaded by two point loads F. The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load F for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 9$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for F.
– Determine the smallest upper-bound for F.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has two simply supported edges (Fig. 3). It carries an evenly distributed <u>line load</u> q [kN/m]. There is no other load on the plate. The plate is homogeneous and orthotropic.

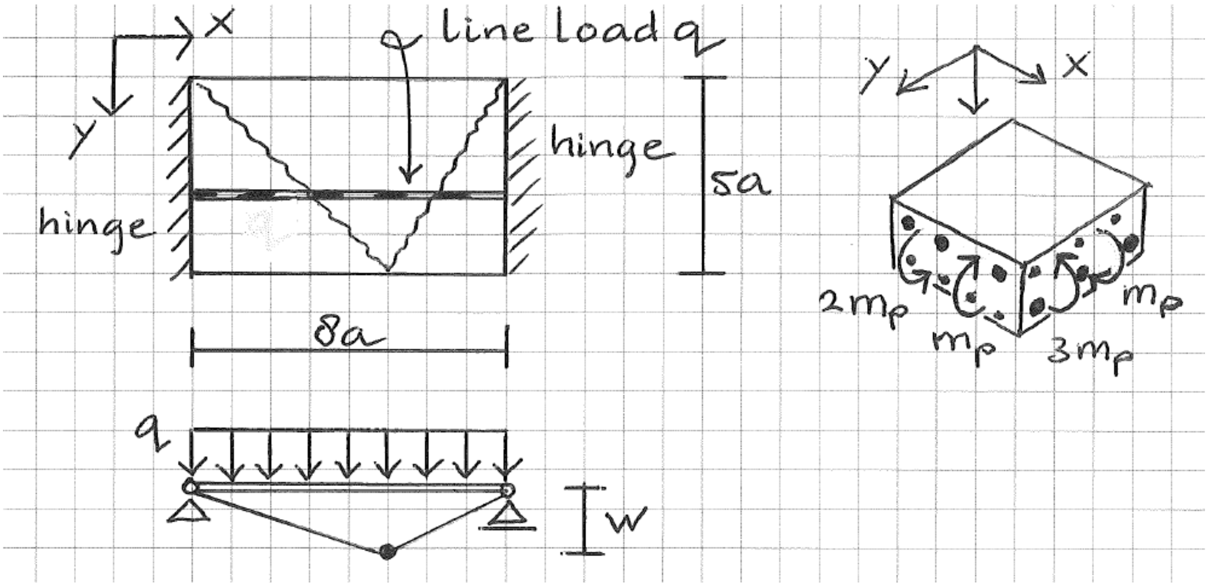


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

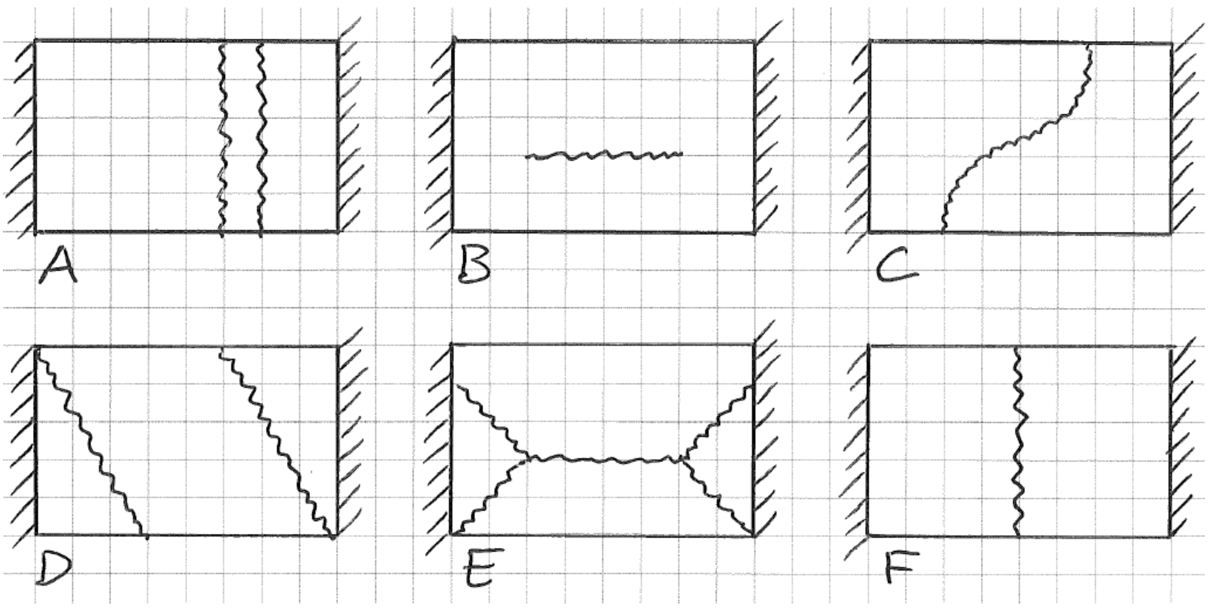


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a A yield line in a steel plate has often a capacity larger than m_p. What causes this? Choose A, B, C or D (0.5 point).

A The deformation has to comply with the normality condition.

B m_p depends on the direction of the yield line.

C Poisson's ratio causes a moment in the other direction too.

D Yielding of steel is described by the Von Mises condition which depends on the moments in all directions and not only on the moment perpendicular to the yield line.

b The evenly distributed load p at which a simply supported circular plate collapses is

$$p = 24 \frac{m_p}{a^2}.$$

Does this solution need top reinforcement? Explain your answer please (0.5 point).

c The following statement has been discussed in class:

Pre-stressing does not change the capacity of a ductile structure.

What is a good defence of this statement? Choose A, B, C or D (0.5 point).

A There is a mathematical proof of this (Pragers 2nd theorem).

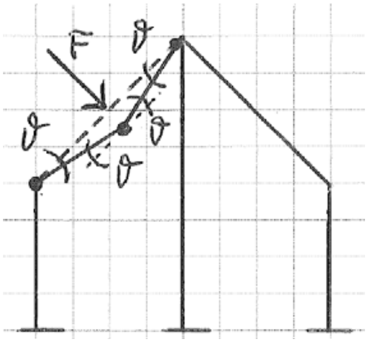
B A plastic mechanism is statically determined.
Statically determined structures cannot be prestressed.

C In a plastic analysis, yielding of a prestressing tendon will remove the prestress.

D It is not always true; for example,
shear failure in prestressed concrete beams can be not ductile.

Answer to problem 1a

```
E:=Mp*t +Mp*(t+t) +Mp*t:
A:=F *t *2*a*sqrt(2):
F:=solve(E=A,F); evalf(F);
```



$$F := \frac{Mp\sqrt{2}}{a}$$

decisive $\boxed{\frac{1.414213562\,Mp}{a}}$

```
E:=Mp*2*t +Mp*2*t + 2*Mp*t + 2*Mp*t + Mp*2*t + Mp*2*t;
```

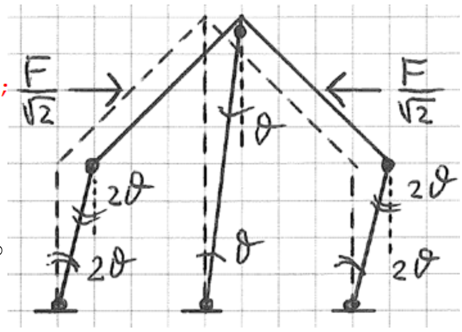
$$E := 12\,Mp\,t$$

```
A:=F/sqrt(2) *2*t *4*a -F/sqrt(2) *2*t *4*a;
```

$$A := 0$$

```
F:=solve(E=A,F); evalf(F);
```

$$F := \frac{12M_p\theta}{0} = \infty$$



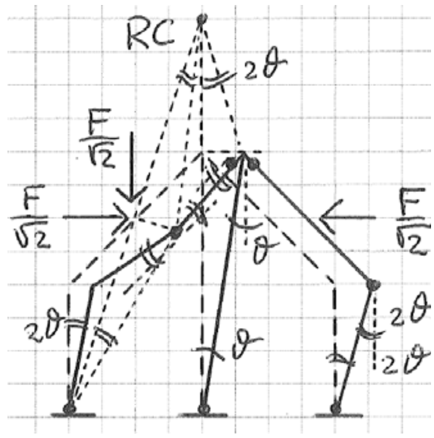

```
E:=Mp*2*t + Mp*(2*t+2*t) +Mp*(2*t+t) + 2*Mp*t +Mp*2*t +Mp*2*t +Mp*t;
```

$$E := 16\,Mp\,t$$

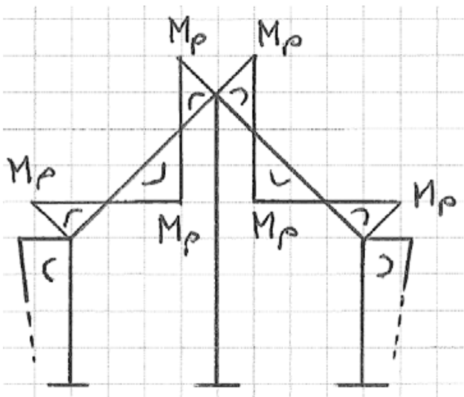
```
A:=F/sqrt(2) *2*t *6*a +F/sqrt(2) *2*t *2*a -F/sqrt(2) *2*t *4*a:
F:=solve(E=A,F); evalf(F);
```

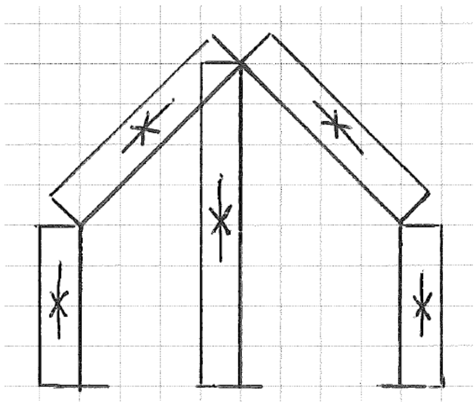
$$F := \frac{2\,Mp\sqrt{2}}{a}$$

$$\frac{2.828427124\,Mp}{a}$$



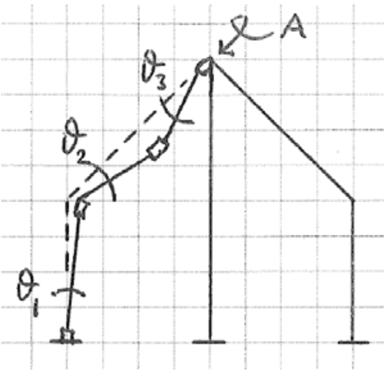
Answer to problem 1b





The moment distribution cannot be drawn completely because it is due to a partial mechanism.
The normal force diagram can only be estimated

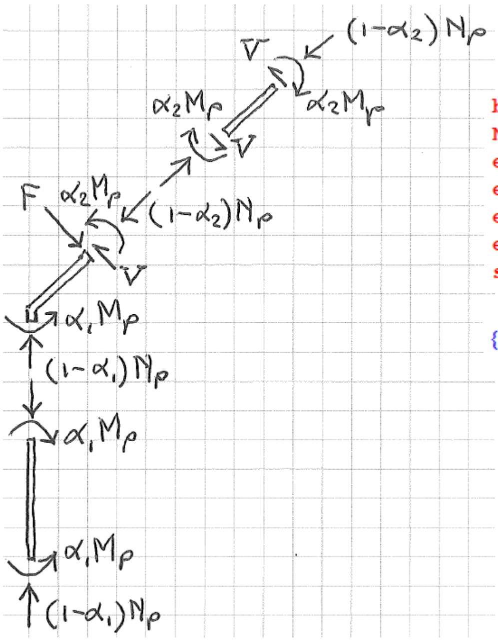
Answer to problem 1c
upperbound




```
beta:=9:
# horzontal displacement of A  (to the right is positive)
eq1:=t1*4*a +t2*2*a -(t2+t3)*a/beta/sqrt(2) -t3*2*a -t3*a/beta/sqrt(2.) = 0:
# vertical displacement of A (down is positive)
eq2:=t1*a/beta +(t2-t1)*a/beta +t2*2*a +(t2+t3)*a/beta/sqrt(2) -t3*2*a +t3*a/beta/sqrt(2) = 0:
E:=Mp*t1 +Mp*(t2-t1) +Mp*(t2+t3) +Mp*t3:
A:=F/sqrt(2) *(t1*4*a +t2*2*a)  +F/sqrt(2) *(t1*a/beta +(t2-t1)*a/beta +t2*2*a):
solve({eq1,eq2,E=A},{t1,t2,F});
```

$$\left\{F = \frac{1.302218088\, Mp}{a},\ t1 = 0.1350073394\, t3,\ t2 = 0.8416144806\, t3\right\}$$

lowerbound




```
beta:=9.:
Np:=beta*Mp/a:
eq1:=F -V -(1-a1)*Np/sqrt(2)=0:
eq2:=a1*Mp +a2*Mp -(1-a1)*Np *2*a=0:
eq3:=(1-a2)*Np = (1-a1)*Np/sqrt(2):
eq4:=V*2*a*sqrt(2) = 2*a2*Mp:
solve({eq1,eq2,eq3,eq4},{F,V,a1,a2});
```

$$\left\{F = \frac{1.302218088\, Mp}{a},\ V = \frac{0.6563636655\, Mp}{a},\ a1 = 0.8985137686,\ a2 = 0.9282383975\right\}$$

Answer to problem 2a

A, D, F

Answer to problem 2b

```
E:=3*mp *5*a *w/(5*a) +mp *5*a *w/(5*a) +3*mp *5*a *w/(3*a) +mp *3*a *w/(5*a);
```

$$E := \frac{48\, mp\, w}{5}$$

```
A:=q*3*a *w*3/10 +q*16/5*a *w*3/5 +q*9/5*a *w*3/10;
```

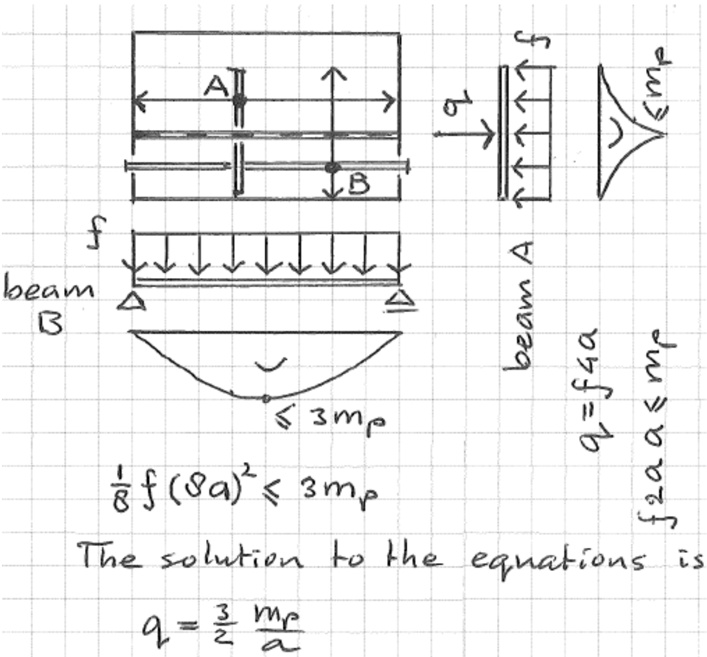
$$A := \frac{84\, q\, a\, w}{25}$$

```
q:=solve(E=A,q); evalf(q);
```

$$q := \frac{20\, mp}{7\, a}$$

$$\frac{2.857142857\, mp}{a}$$

Answer to problem 2c



Answer to problem 3

A

No, all yield lines open at the bottom of the plate.

C