Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 12 April 2018, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

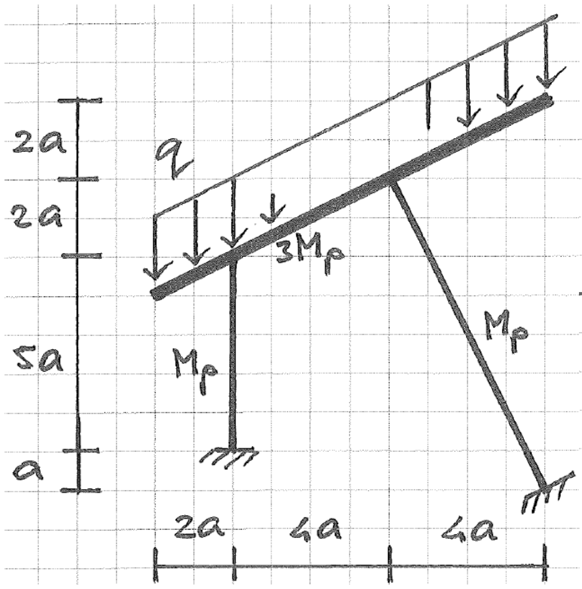


Figure 1. Frame structure

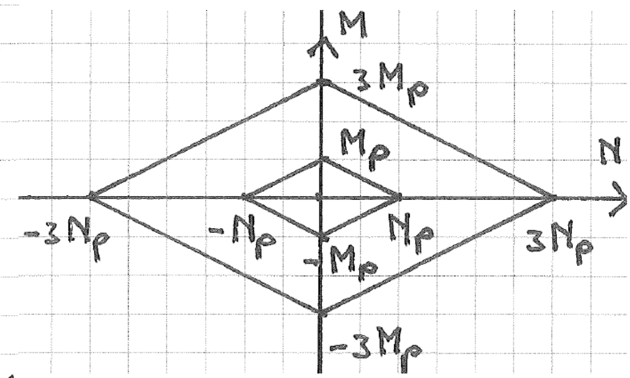


Figure 2. Yield contours

Problem 1

A frame consists of two columns and a beam (Fig.1) The columns have strength M_p, the beam has strength $3M_p$. All elements are rigidly connected. The beam is loaded by an evenly distributed line load q. The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 8. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate is simply supported at three places (Fig. 3). It carries an evenly distributed load p [kN/m^2] in the z direction. There is no other load on the plate. The plate is homogeneous and orthotropic.

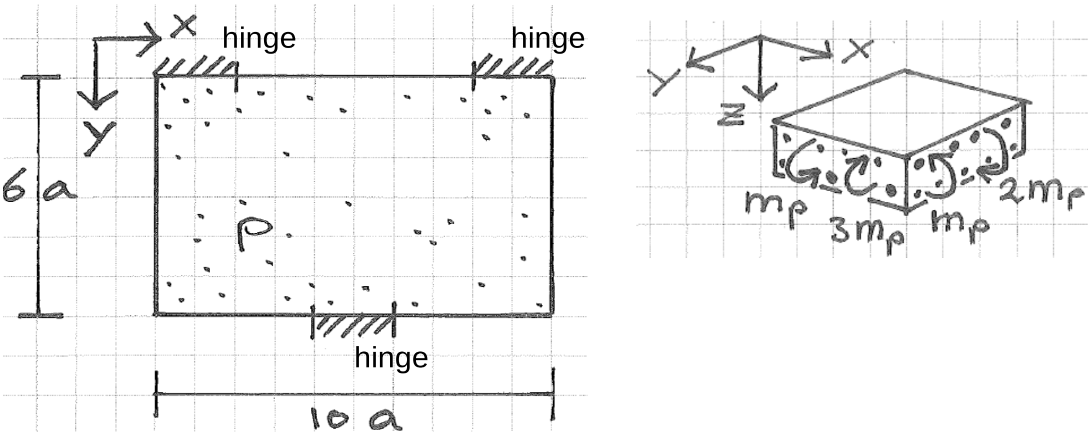


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

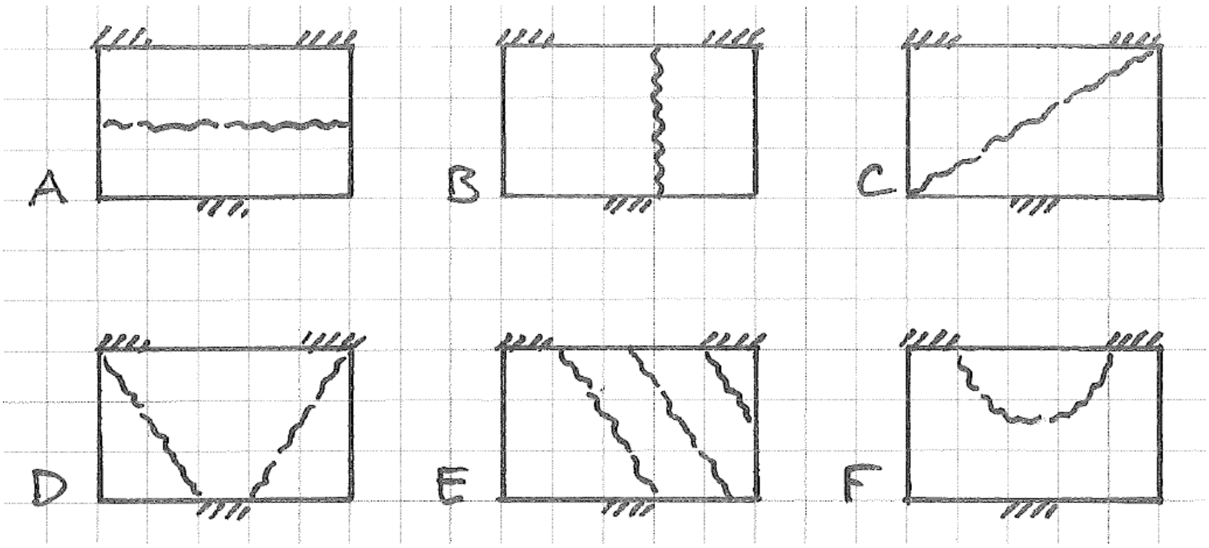


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an upper bound for p expressed in m_p and a (1.5 point).

c Determine the largest lower-bound for p using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

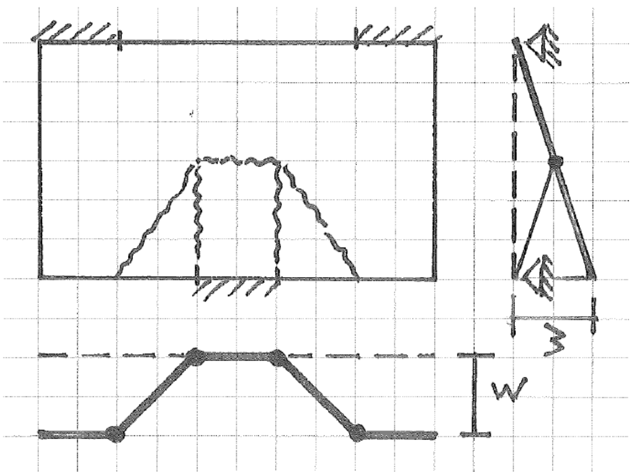


Figure 5. Mechanism of problem 2b

Problem 3

a A circular plate is simply supported on its edge. The plate diameter is *a*. The plate strength is m_p in all directions. The load *p* is perpendicular to the plate and evenly distributed. The collapse load is … Choose A, B, C or D. (0.5 points)

A $p = 48\dfrac{m_p}{a^2}$

B $p = 24\dfrac{m_p}{a^2}$

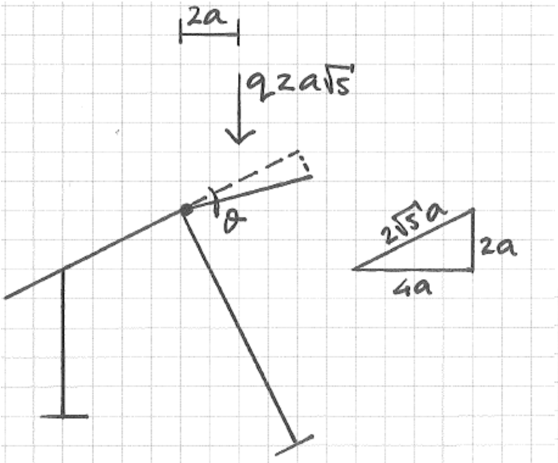
C $p \le 24\dfrac{m_p}{a^2}$

D $p = 4\pi\dfrac{m_p}{a^2}$

b A frame structure is statically indetermined to the 7th degree. It has 13 locations where plastic hinges could occur. In an upper bound analysis we study the mechanisms. How many mechanisms should be considered in theory? (0.5 points)

c Put the following three words in the order in which they happen in time. (0.5 points)
- cable action;
- plastic limit load;
- arch action or dome effect.

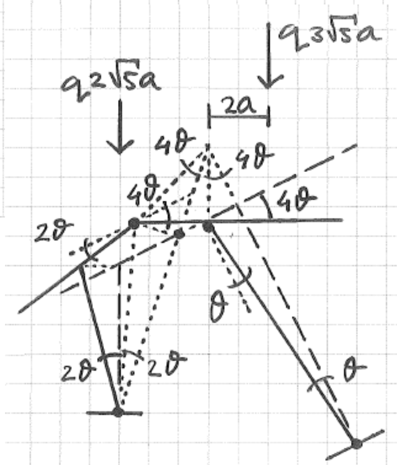
Answer to Problem 1a



$$E = 3 M_p \vartheta$$

$$A = q\, 2\sqrt{5} a\ \vartheta\ 2a$$

$$E = A \Rightarrow q = \frac{3}{4\sqrt{5}} \frac{M_p}{a^2} \qquad 0.335$$



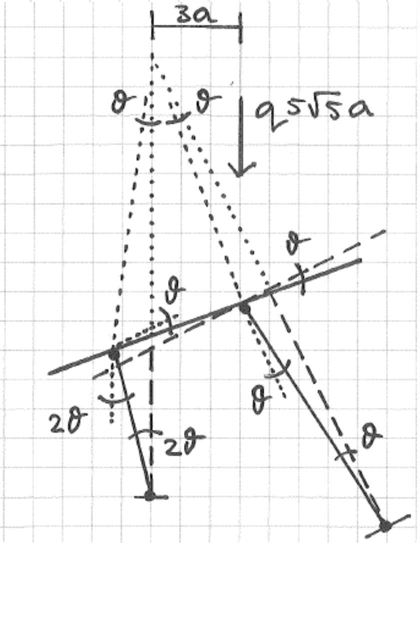
$$E = M_p 2\vartheta + 3M_p(2\vartheta + 4\vartheta) + M_p(4\vartheta + \vartheta) + M_p \vartheta$$

$$= 26 M_p \vartheta$$

$$A = q\, 3\sqrt{5} a\ 4\vartheta\ 2a$$

$$= 24\sqrt{5}\, q a^2 \vartheta$$

$$E = A \Rightarrow q = \frac{13}{12\sqrt{5}} \frac{M_p}{a^2} \qquad 0.484$$



$$E = M_p 2\vartheta + M_p(2\vartheta + \vartheta) + M_p(\vartheta + \vartheta) + M_p \vartheta$$

$$= 8 M_p \vartheta$$

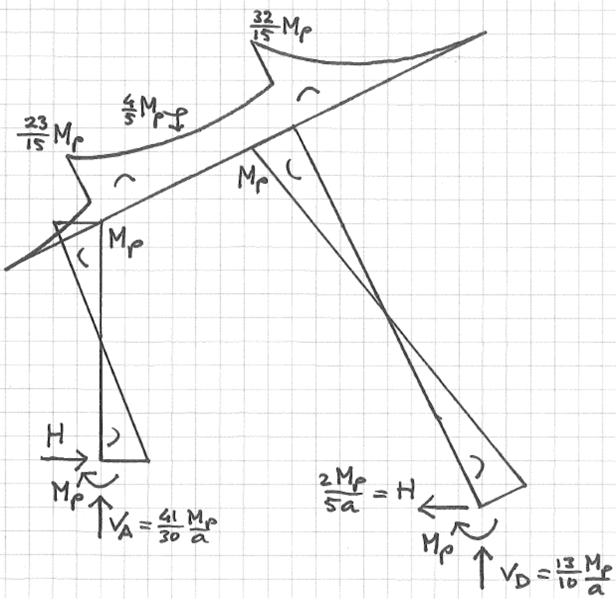
$$A = q\, 5\sqrt{5} a\ \vartheta\ 3a$$

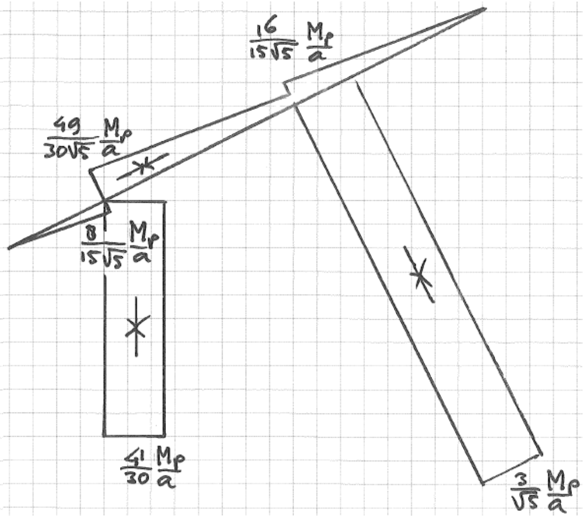
$$= 15\sqrt{5}\, q a^2 \vartheta$$

$$E = A \Rightarrow \boxed{q = \frac{8}{15\sqrt{5}} \frac{M_p}{a^2}} \qquad 0.239$$

decisive

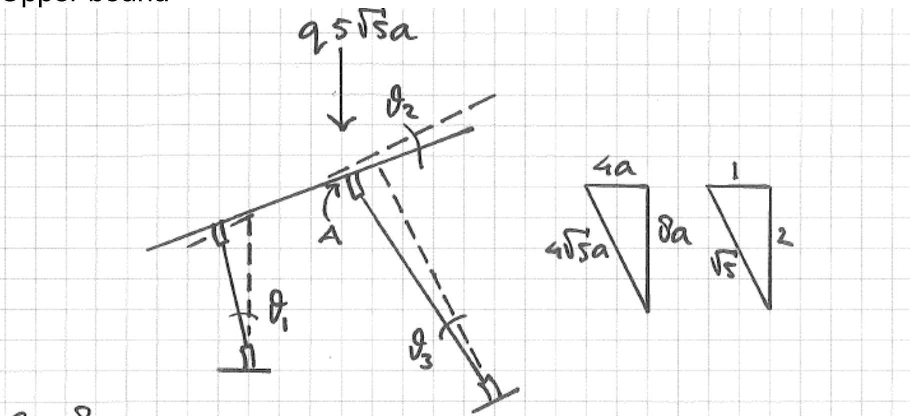
Answer to Problem 1b





Answer to Problem 1c

Upper bound



$\beta = 8$

hor. displ. of A ← +

$$\vartheta_1 5a - \vartheta_2 2a = \vartheta_3 8a - \vartheta_3 \frac{a}{\beta}\frac{1}{\sqrt{5}} - (\vartheta_3 + \vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{5}}$$

vert. displ. of A ↓ +

$$\vartheta_1 \frac{a}{\beta} + (\vartheta_1 + \vartheta_2)\frac{a}{\beta} + \vartheta_2 4a = \vartheta_3 4a + \vartheta_3 \frac{a}{\beta}\frac{2}{\sqrt{5}} + (\vartheta_3 + \vartheta_2)\frac{a}{\beta}\frac{2}{\sqrt{5}}$$

$$E = M_p \vartheta_1 + M_p(\vartheta_1 + \vartheta_2) + M_p(\vartheta_2 + \vartheta_3) + M_p \vartheta_3$$

$$A = q\, 5\sqrt{5}a \left(\vartheta_1 \frac{a}{\beta} + (\vartheta_1 + \vartheta_2)\frac{a}{\beta} + \vartheta_2\, 3a\right)$$

$$E = A$$

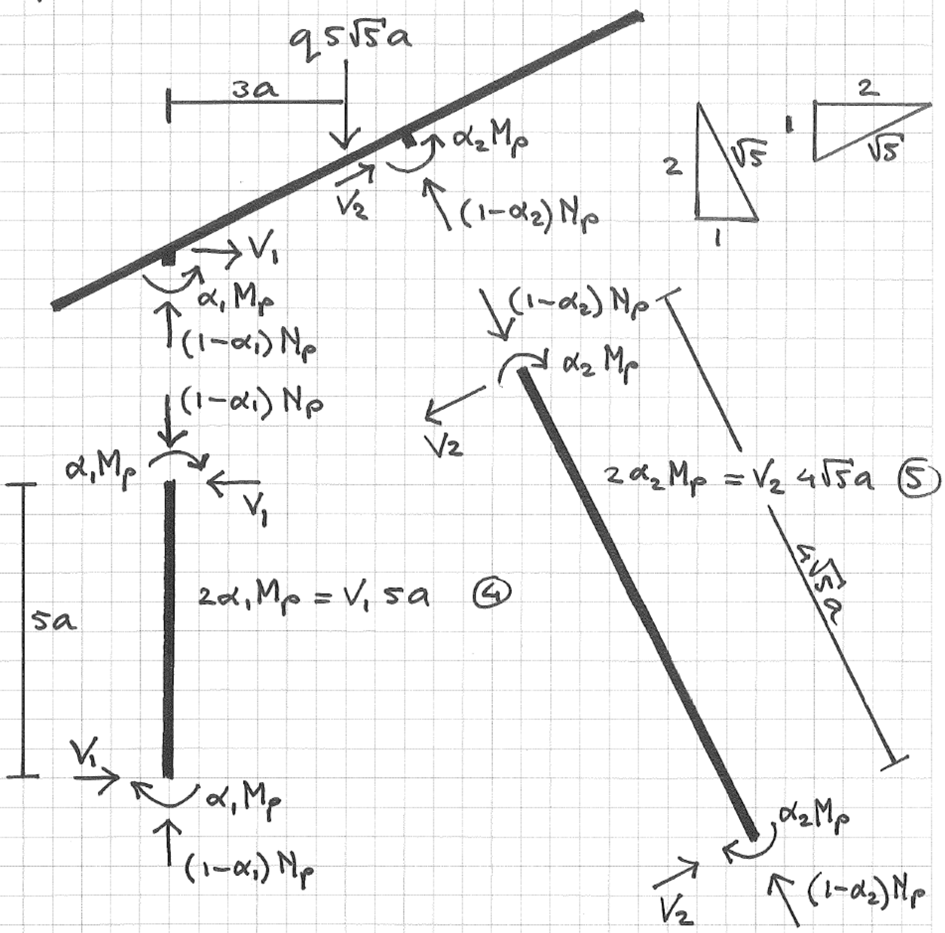
The solution to the equations is $q = 0.2039 \frac{M_p}{a^2}$ $\quad \theta_1 = 2.08\,\theta_2 \quad \theta_3 = 1.07\,\theta_2$

Lower bound

$$q\,5\sqrt{5}a - (1-\alpha_1)N_p - V_2\frac{1}{\sqrt{5}} - (1-\alpha_2)N_p\frac{2}{\sqrt{5}} = 0 \quad (1)$$

$$V_1 + V_2\frac{2}{\sqrt{5}} - (1-\alpha_2)N_p\frac{1}{\sqrt{5}} = 0 \quad (2)$$

$$\alpha_1 M_p + \alpha_2 M_p - q\,5\sqrt{5}a\;3a + (1-\alpha_2)N_p\,2\sqrt{5}a = 0 \quad (3)$$



5 equations from which can be solved

$V_1 \quad V_2 \quad \alpha_1 \quad \alpha_2 \quad q$

The solution to the equations is

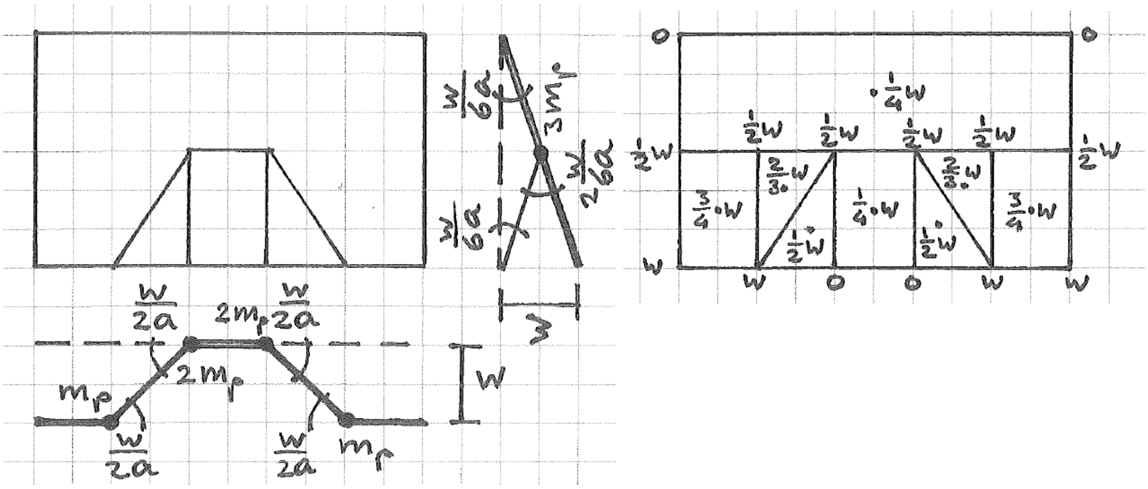
$$q = 0.2039\frac{M_p}{a^2} \quad \alpha_1 = 0.854 \quad \alpha_2 = 0.857 \quad V_1 = 0.342\frac{M_p}{a} \quad V_2 = 0.192\frac{M_p}{a}$$

Answer to Problem 2a

A, D, E

0 wrong = 1.0 point
1 wrong = 0.7 point
2 wrong = 0.3 point
3 wrong = 0.0 point

Answer to Problem 2b



$$E = 3m_p\,2a\,\frac{w}{3a} + \left(m_p\,3a\,\frac{w}{2a} + 3m_p\,2a\,\frac{w}{3a}\right)2 + \left(2m_p\,3a\,\frac{w}{2a}\right)2$$

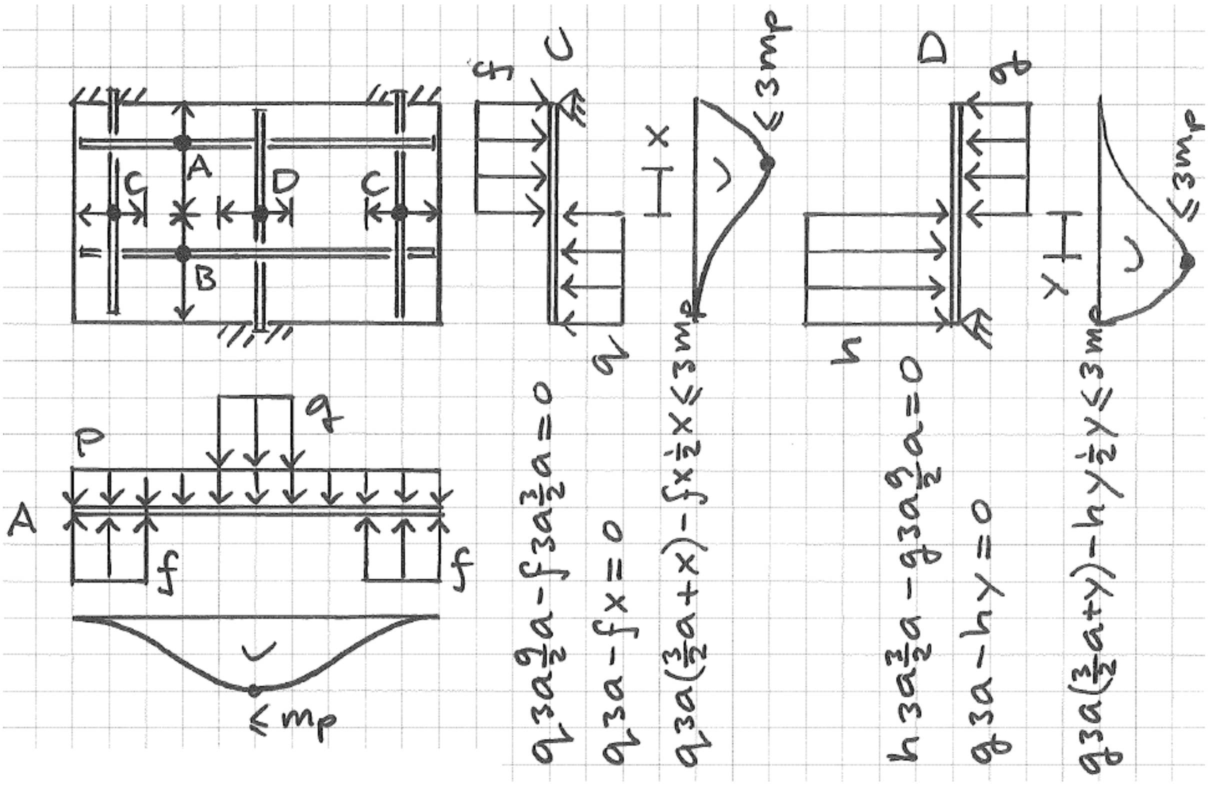
$$= 15\,m_p w$$

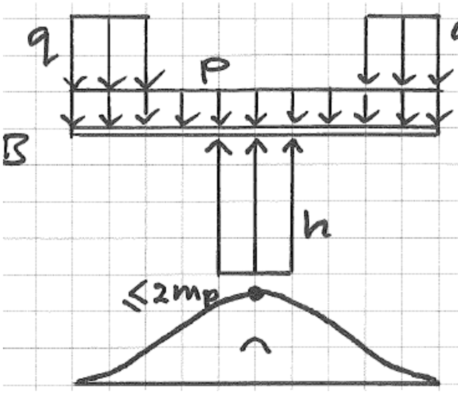
$$A = p\,10a\,3a\,\tfrac{1}{4}w + p\,2a\,3a\,\tfrac{3}{4}w\,2 + p\,\tfrac{1}{2}\,2a\,3a\,\tfrac{2}{3}w\,2 + p\,\tfrac{1}{2}\,2a\,3a\,\tfrac{1}{2}w\,2 + p\,2a\,3a\,\tfrac{1}{4}w$$

$$= 25\,p a^2 w$$

$$E = A \Rightarrow p = \frac{3}{5}\frac{m_p}{a^2} \qquad 0{,}6$$

Answer to Problem 2c





$$q\,2a + p\,10a - f\,4a = 0$$

$$f\,2a\,4a - p\,5a\,\tfrac{5}{2}a - q\,a\,\tfrac{1}{2}a \leq m_p$$

$$q\,4a + p\,10a - h\,2a = 0$$

$$q\,2a\,4a + p\,5a\,\tfrac{5}{2}a - h\,a\,\tfrac{1}{2}a \leq 2m_p$$

The solution to the equations is $p = \frac{4}{65}\frac{m_p}{a^2}$ 0.06

Answer to Problem 3a
B (see reader Plates, page 40)

Answer to Problem 3b
$$\frac{13!}{8!(13-8)!} = 1287$$

Answer to Problem 3c
plastic limit load; arch action; cable action (see reader Frames, page 94)