Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 25 January 2018, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

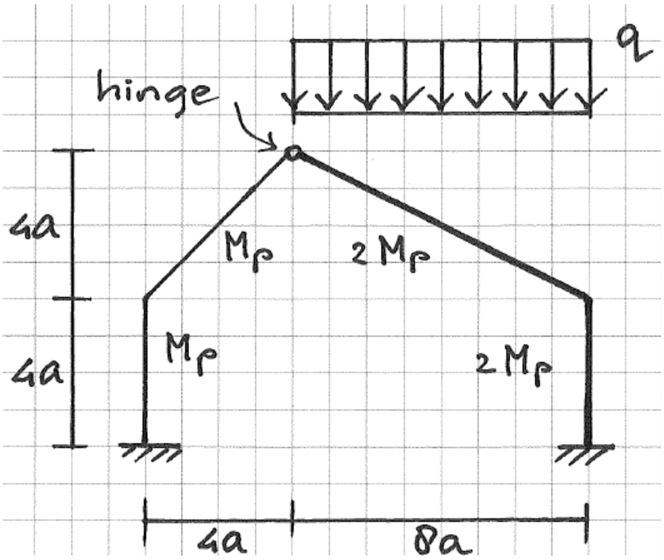


Figure 1. Frame structure

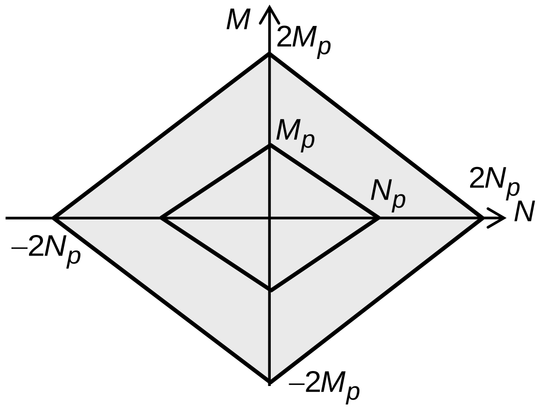


Figure 2. Yield contours

Problem 1

A frame consists of two columns and two beams (Fig.1) The right two elements have a strength $2M_p$, the left two elements have a strength M_p. All elements are rigidly connected except for the beams which are connected together by a hinge. The structure is loaded by an evenly distributed line load q (snow). The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 10. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has three simply supported edges and three free edges (Fig. 3). It carries an evenly distributed load p [kN/m^2]. There is no other load on the plate. The plate is homogeneous and orthotropic.

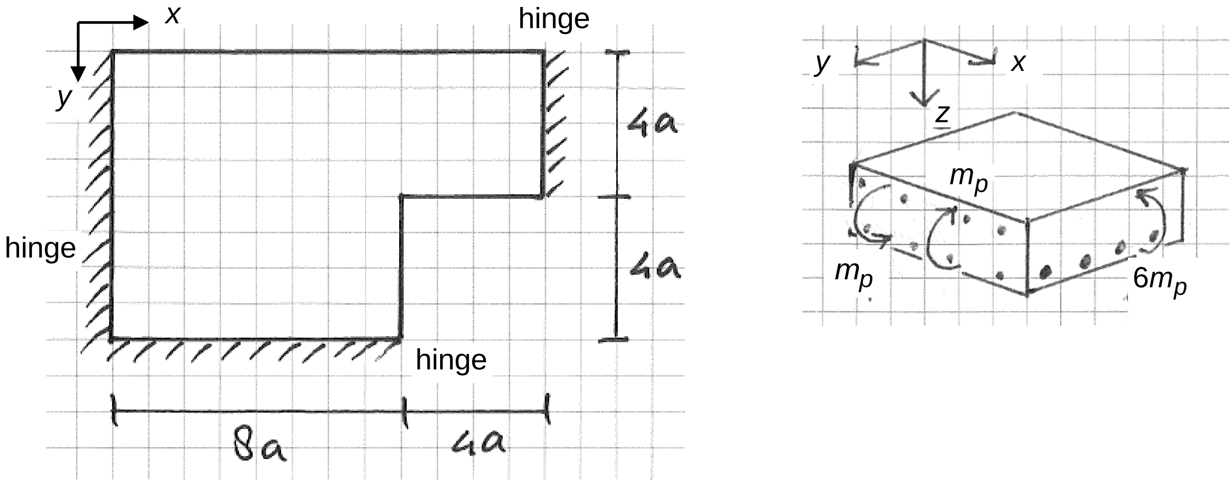


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

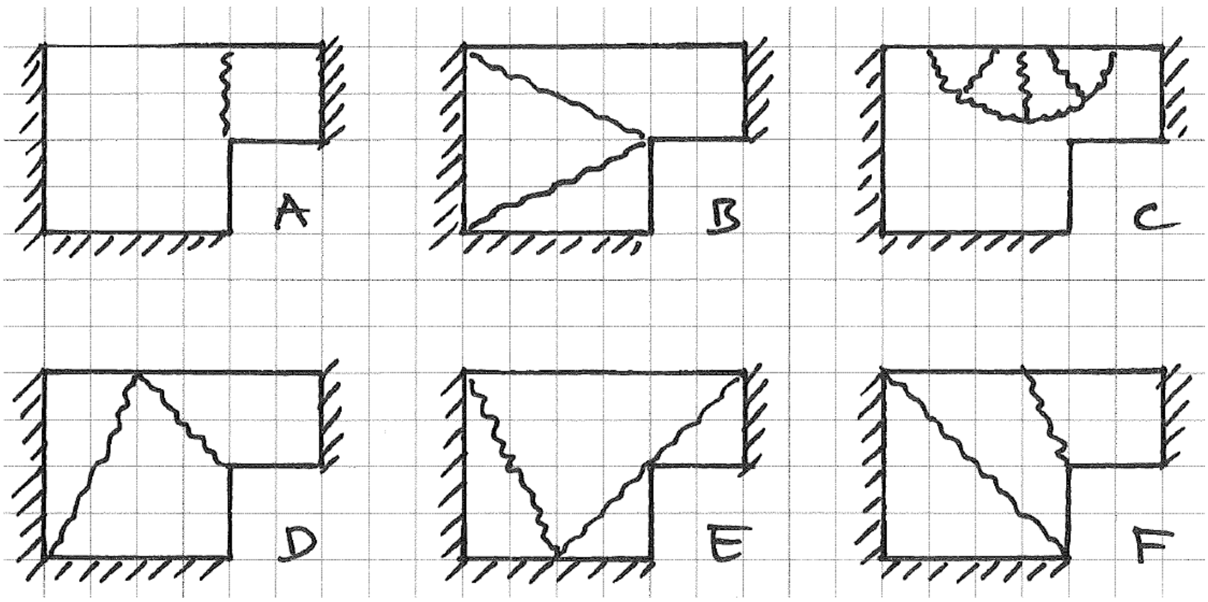


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for p expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for p using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

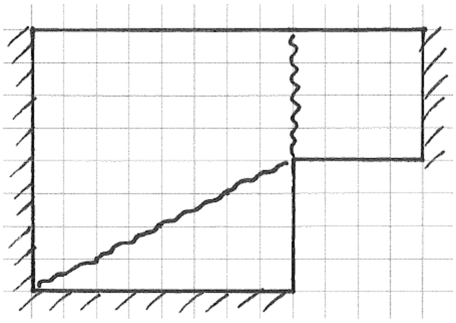


Figure 5. Mechanism of problem 2b

Problem 3

a A square plate is simply supported on two adjacent edges. The plate length and width are a. The plate strength is m_p in all directions. The load is perpendicular to the plate and evenly distributed. The collapse load is … Choose A, B, C or D. (0.5 points)

A $6\dfrac{m_p}{a^2}$

B $5.55\dfrac{m_p}{a^2}$

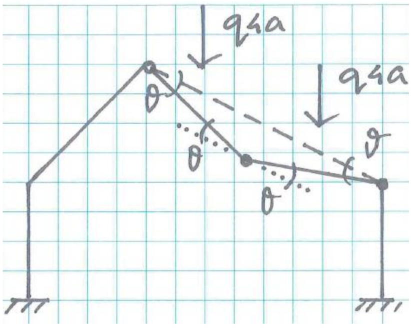
C $5.50\dfrac{m_p}{a^2}$

D $\leq 5.50\dfrac{m_p}{a^2}$

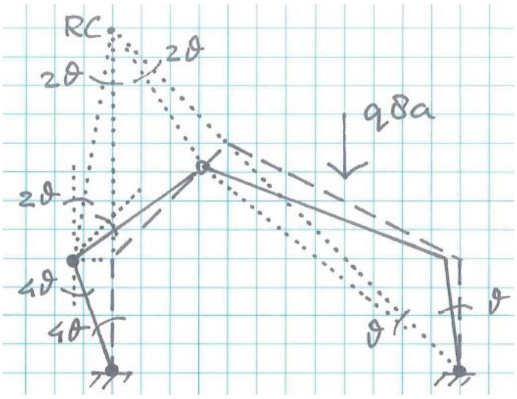
b Shear force has little influence on the moment capacity of steel I sections.
In which figure of the lecture book is this convincingly shown? (0.5 points)

c A frame structure can have a symmetrical geometry and symmetrical loading. However, the collapse mechanism can be asymmetrical. Give an example of this. (0.5 points)

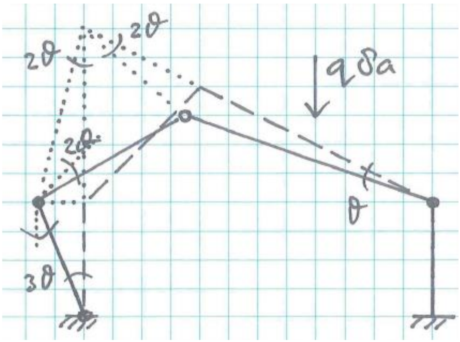
Answer to Problem 1a



$$E = 2M_p(\vartheta+\vartheta) + 2M_p\vartheta$$
$$A = q4a\,\vartheta\,2a + q4a\,\vartheta\,2a$$
$$E = A \Rightarrow q = \frac{3}{8}\frac{M_p}{a^2}$$

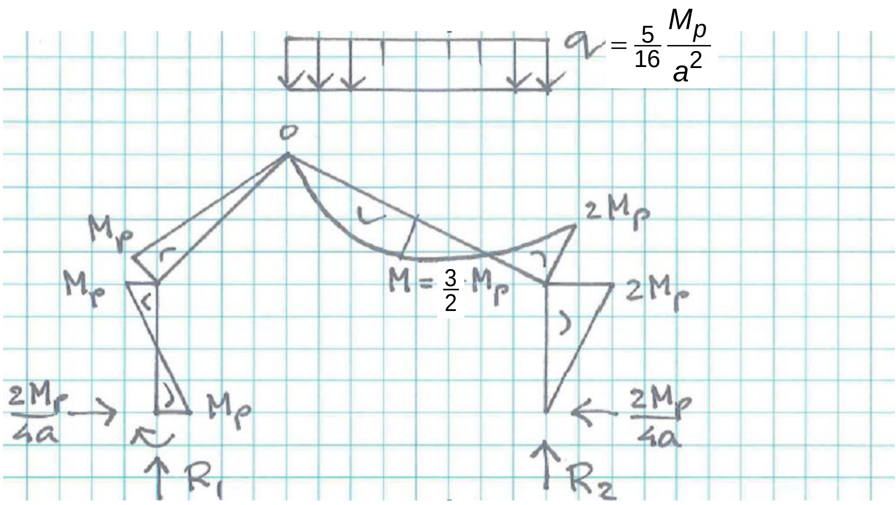


$$E = M_p 4\vartheta + M_p(4\vartheta+2\vartheta) + 2M_p\vartheta$$
$$A = q\,8a\,\vartheta\,4a$$
$$E = A \Rightarrow q = \frac{3}{8}\frac{M_p}{a^2}$$



$$E = M_p 3\vartheta + M_p(3\vartheta+2\vartheta) + 2M_p\vartheta$$
$$A = q\,8a\,\vartheta\,4a$$
$$E = A \Rightarrow \boxed{q = \frac{5}{16}\frac{M_p}{a^2}} \quad \text{decisive}$$

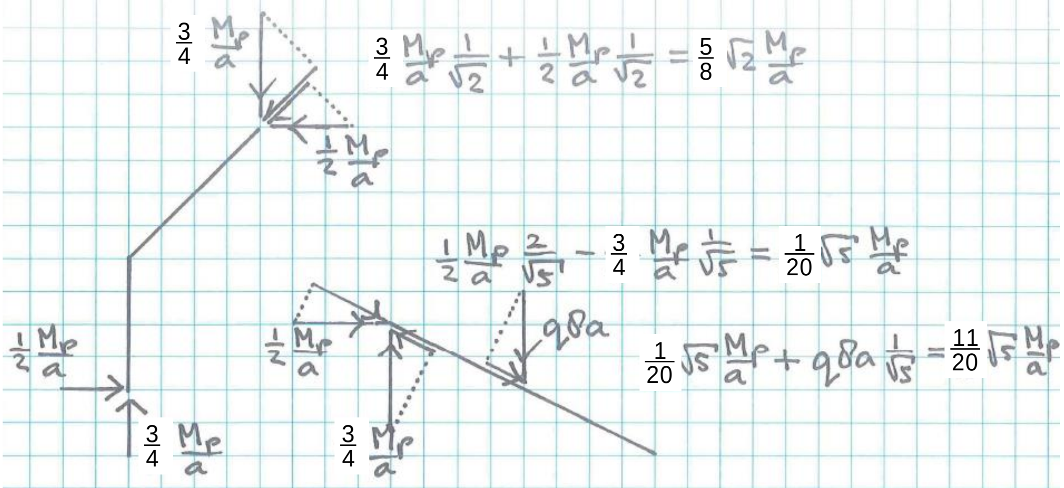
Answer to Problem 1b

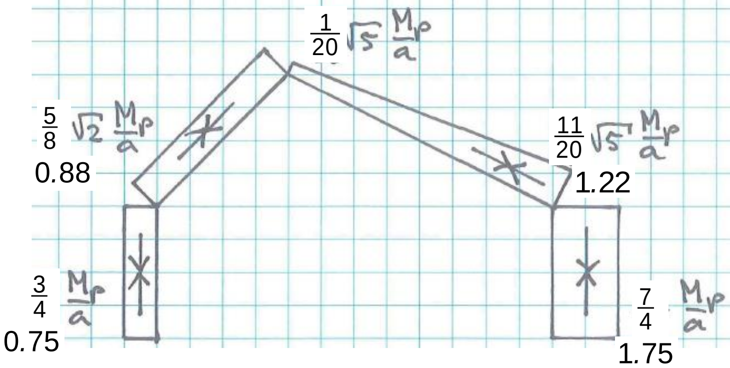


$$M_p + R_1 4a - \frac{2M_p}{4a} 8a = 0 \quad \Rightarrow \quad R_1 = \frac{3}{4}\frac{M_p}{a}$$

$$R_1 + R_2 = q\,8a \quad \Rightarrow \quad R_2 = \frac{7}{4}\frac{M_p}{a}$$

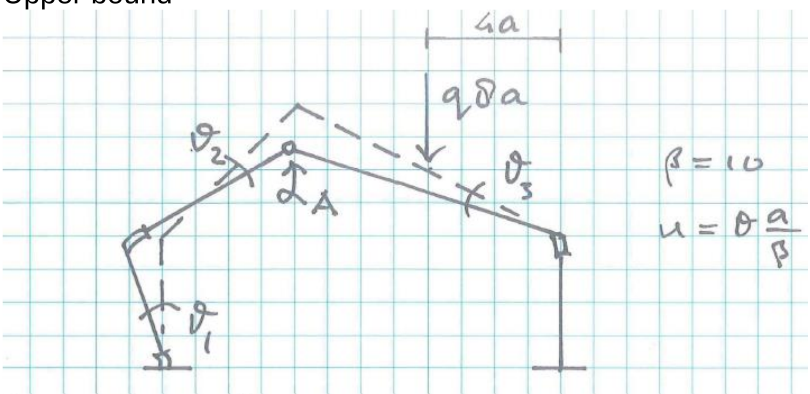
$$M = R_2 4a - \frac{2M_p}{4a} 6a - q\,4a\,2a = \frac{3}{2} M_p$$





Answer to Problem 1c

Upper bound



hor. disp. A →

$$-\vartheta_1 4a - (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_2 4a = -\vartheta_3 4a$$

vert. disp. A ↓

$$\vartheta_1\frac{a}{\beta} + (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_2 4a = \vartheta_3\frac{a}{\beta} + \vartheta_3 8a$$

$$E = M_p \vartheta_1 + M_p(\vartheta_1+\vartheta_2) + 2M_p \vartheta_3$$

$$A = q\,8a\left(\vartheta_3 \frac{a}{\beta} + \vartheta_3 4a\right)$$

$$E = A$$

The solution to the equations is

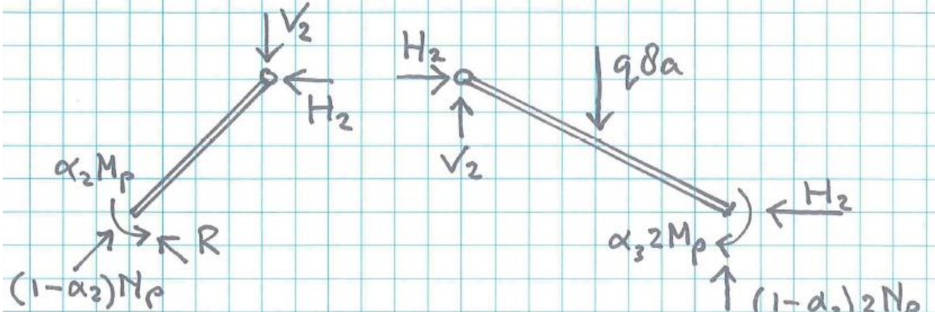
$$q = 0{,}288\,\frac{M_p}{a^2}\ ,\quad \vartheta_1 = 2{,}79\,\vartheta_3\ ,\quad \vartheta_2 = 1{,}87\,\vartheta_3$$

Lower bound

$$\frac{V_2}{\sqrt{2}} - \frac{H_2}{\sqrt{2}} - R = 0 \qquad q\,8a - V_2 - (1-\alpha_3)\,2N_p = 0$$

$$\frac{V_2}{\sqrt{2}} + \frac{H_2}{\sqrt{2}} = (1-\alpha_2)N_p \qquad q\,8a\,4a - (1-\alpha_3)\,2N_p\,8a + \alpha_3 2M_p +$$

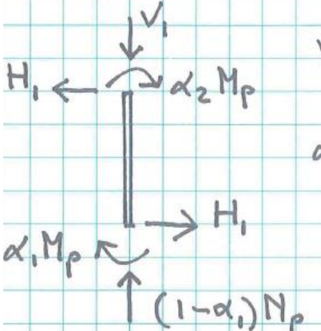
$$\alpha_2 M_p - R\,4a\sqrt{2} = 0 \qquad + H_2 4a = 0$$



$$\frac{V_1}{\sqrt{2}} - \frac{H_1}{\sqrt{2}} = R$$

$$\frac{V_1}{\sqrt{2}} + \frac{H_1}{\sqrt{2}} = (1-\alpha_2)N_p \qquad \beta = 10$$

$$N_p = \beta\,\frac{M_p}{a}$$



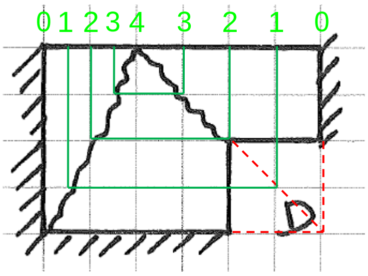
$$V_1 = (1-\alpha_1)N_p$$

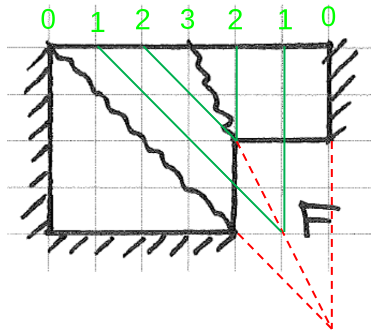
$$\alpha_2 M_p + \alpha_1 M_p = H_1 4a$$

The solution to the equations is

$$q = 0.288\,\frac{M_p}{a^2}\ ,\quad \alpha_1 = 0{,}931\ ,\quad \alpha_2 = 0{,}918 \quad \alpha_3 = 0{,}919$$

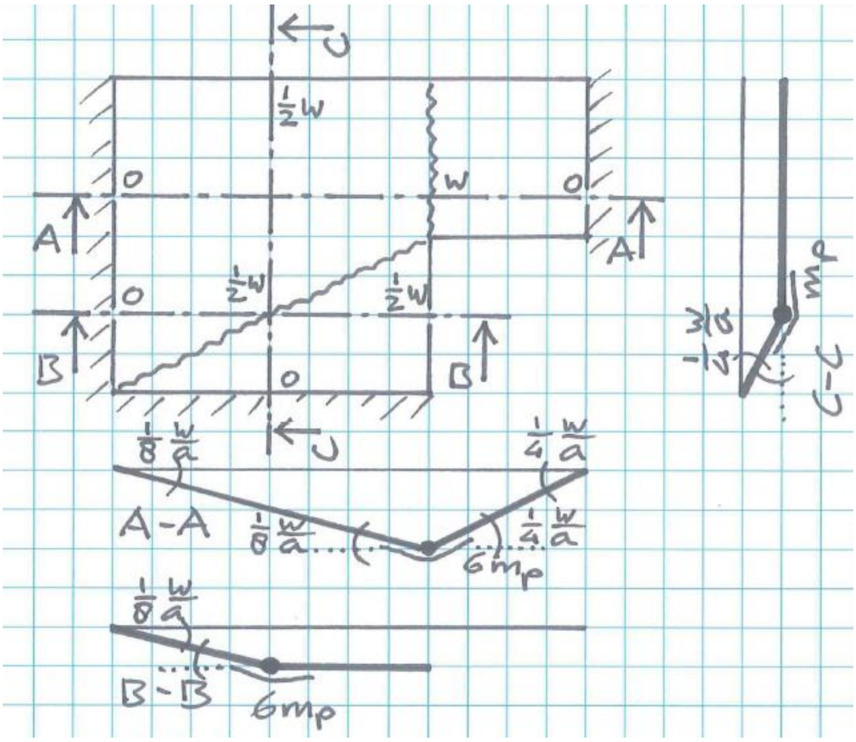
Answer to Problem 2a
C, D, F





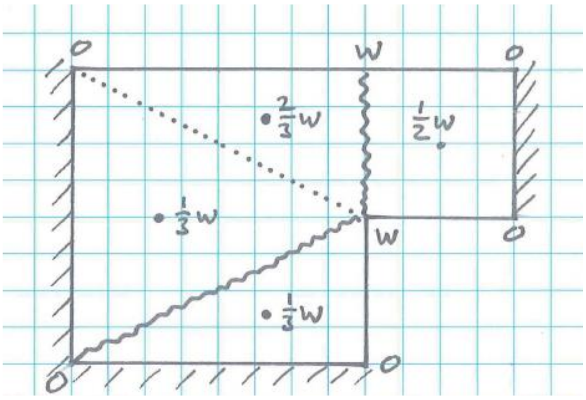
(The figures are not required)

Answer to Problem 2b



$$E = 6m_p\,4a\left(\frac{1}{8}\frac{w}{a} + \frac{1}{4}\frac{w}{a}\right) + 6m_p\,4a\,\frac{1}{8}\frac{w}{a} + m_p\,8a\,\frac{1}{4}\frac{w}{a}$$

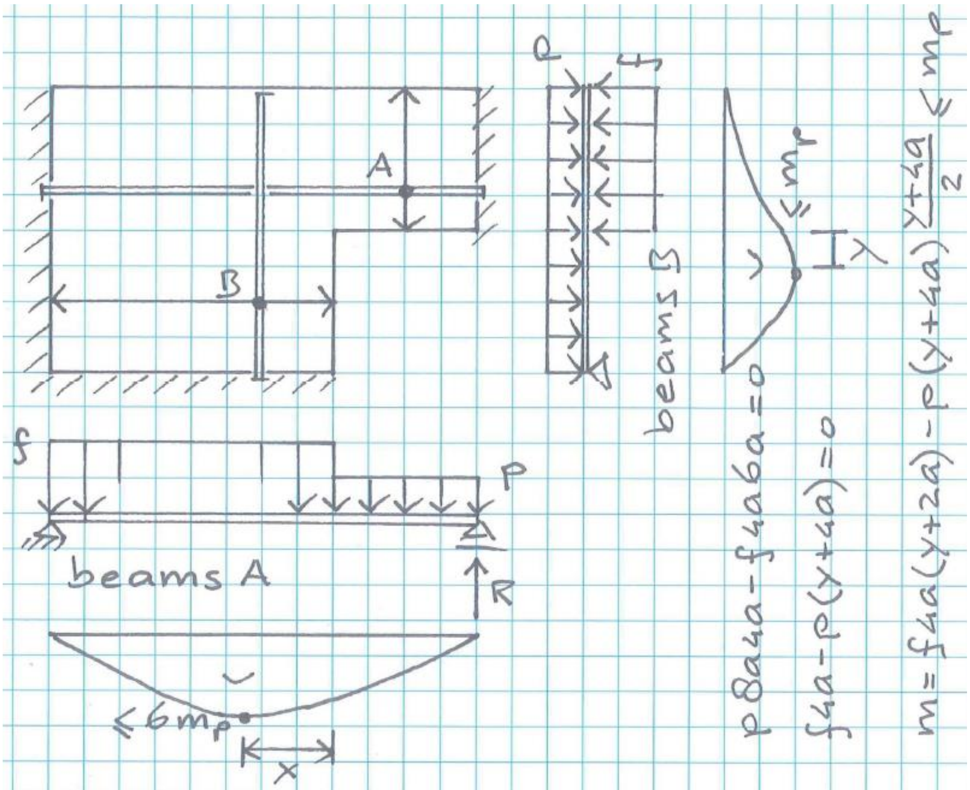
$$= 14\,m_p w$$



$$A = p\frac{8a\,4a}{2}\frac{1}{3}w + p\frac{8a\,8a}{2}\frac{1}{3}w + p\frac{8a\,4a}{2}\frac{2}{3}w + p\,4a\,4a\frac{1}{2}w$$
$$= \frac{104}{3}\,p a^2 w$$

$$E = A \Rightarrow \quad p = \frac{21}{52}\frac{m_p}{a^2} = 0{,}40\frac{m_p}{a^2}$$

Answer to Problem 2c



$$R\,12a - p\,4a\,10a - f\,8a\,4a = 0$$
$$R - p\,4a - f x = 0$$
$$m = R(x+4a) - p\,4a(x+2a) - f x\frac{x}{2} \leq 6m_p$$

The solution to the equations is $p = \frac{324}{1225}\frac{m_p}{a^2} = 0.26\frac{m_p}{a^2}$.

Answer to Problem 3a
D

Answer to Problem 3b
Fig. 8.8

Answer to Problem 3c
Exam August 2005