Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 22 January 2019, 13:30 – 16:30 hours

Write your <u>name</u> and <u>study number</u> at the top of your work.

Also write whether you were a <u>member</u> of the elastic team, plastic team or no team.

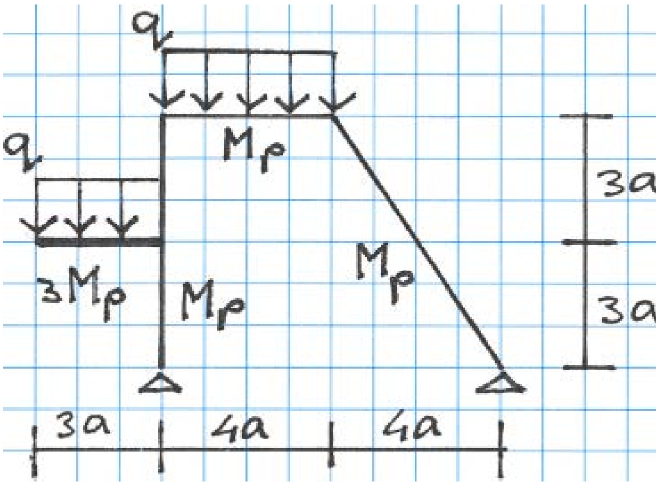


Figure 1. Frame structure

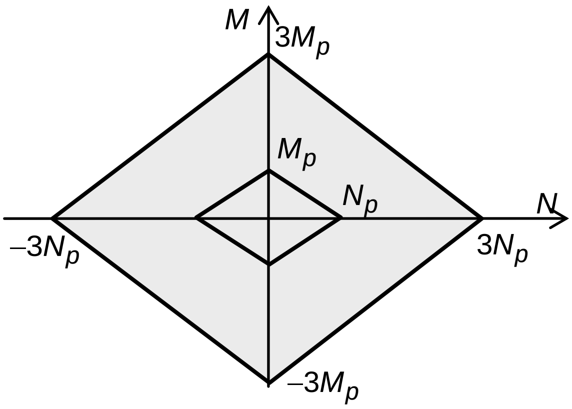


Figure 2. Yield contours

Problem 1

A frame consists of two columns, a beam and a cantilever (Fig.1). The elements have a strength M_p except for the cantilever, which has a strength $3M_p$. The elements are rigidly connected. The support consist of two hinges. The structure is loaded by two evenly distributed line loads q. The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 12. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for q.
– Determine the smallest <u>upper-bound</u> for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has simply supported edges and free edges (Fig. 3). It carries an evenly distributed load p [kN/m^2]. There is no other load on the plate. The plate is homogeneous and orthotropic.

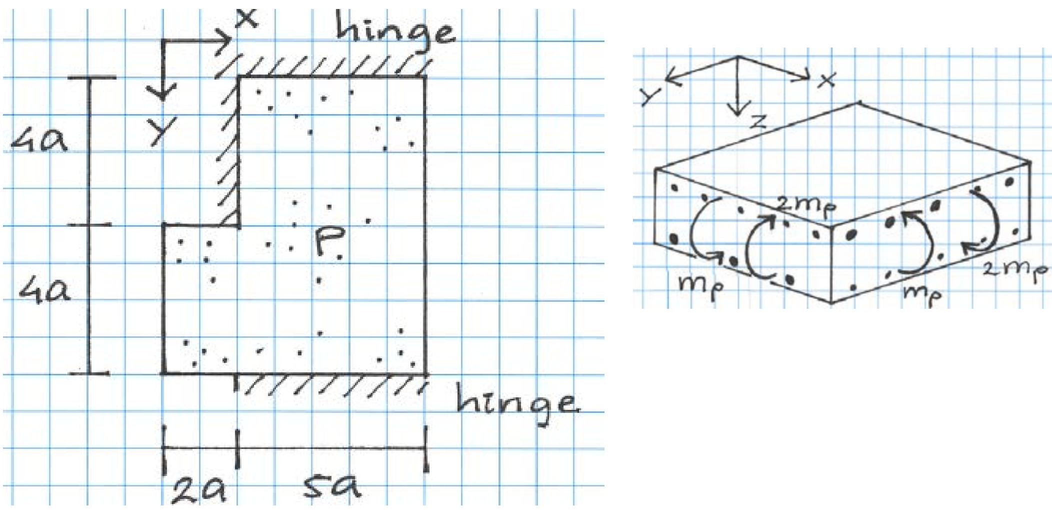


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

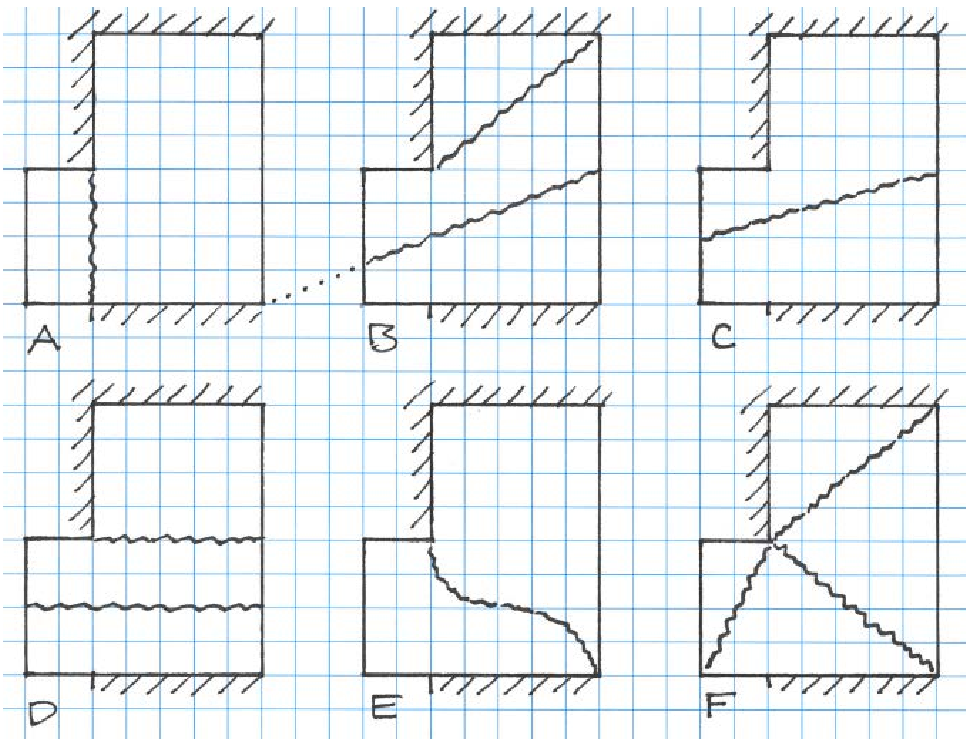


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for *p* expressed in m_p and *a* (1.5 point).

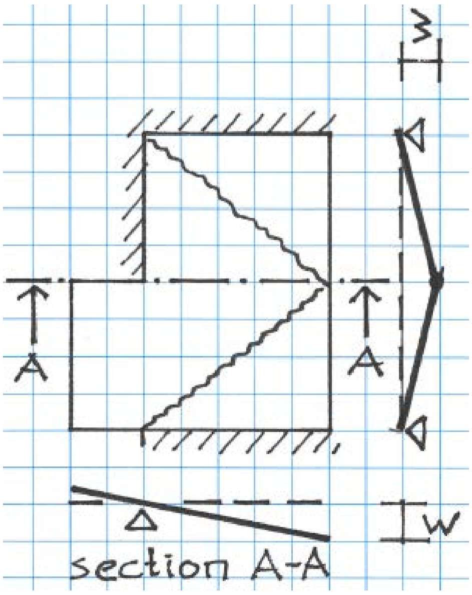


Figure 5. Mechanism of problem 2b

c Determine the largest <u>lower-bound</u> for *p* using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

Problem 3

a A plate is supported in a 90^o corner point. The support is a hinge. The support reaction is perpendicular to the plate. The plate strength is m_p in all directions. The collapse load is …
Choose A, B, C or D. (0.5 points)

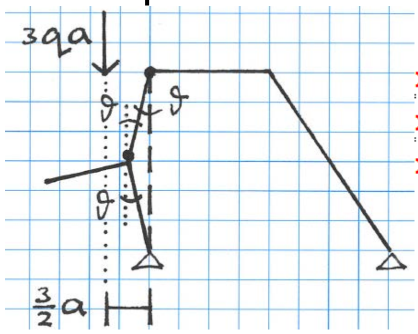
A πm_p
B $2m_p$
C $\leq 2m_p$
D 0 because it is a singularity

b In limit analysis, how do we know whether the exact plastic strength has been found?
Choose A, B, C or D. (0.5 points)

A When the lower- and upper-bound are the same.
B When all possible mechanisms have been considered.
C When it has been confirmed by finite element analysis.
D When Prager's theorems can be proofed.

c Which yield contour is most suitable for modelling metals?
How do we know this is true? (0.5 points)

Answer to problem 1a




```
> E:=Mp*t +Mp*(t+t):
> A:=q*3*a*t*3/2*a:
> q:=solve(E=A,q);evalf(q);
```

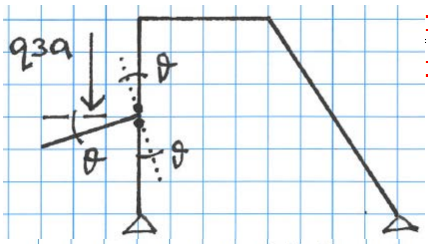
$$q := \frac{2}{3}\frac{Mp}{a^2}$$

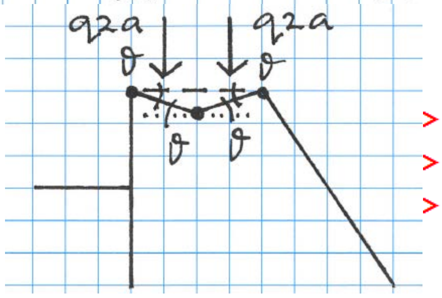
$$\frac{0.6666666667\,Mp}{a^2}$$

```
> E:=Mp*t +Mp*t:
> A:=q*3*a*t*3/2*a:
> q:=solve(E=A,q);evalf(q);
```

$$\frac{4}{9}\frac{Mp}{a^2}$$

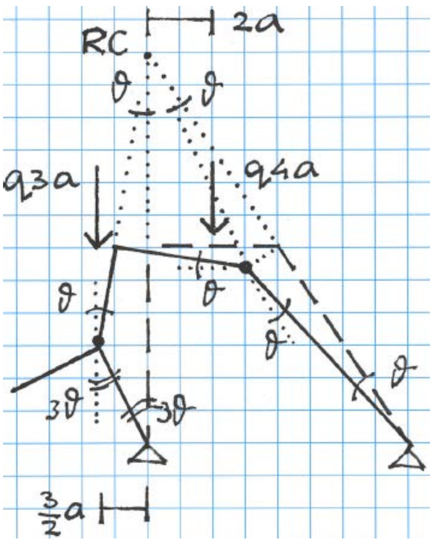
$$\frac{0.4444444444\,Mp}{a^2}$$

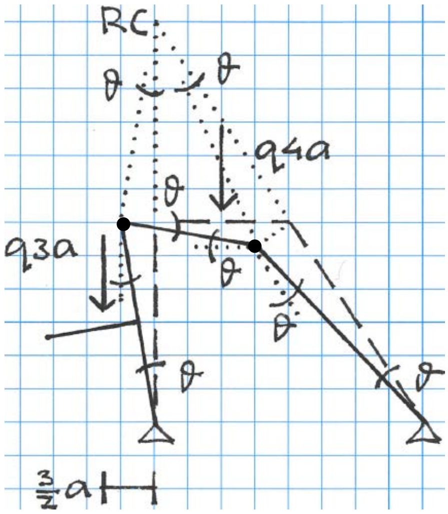





```
> E:=Mp*t +Mp*(t+t) +Mp*t:
> A:=q*2*a*t*a +q*2*a*t*a:
> q:=solve(E=A,q);
```

$$\frac{Mp}{a^2}$$






```
> E:=Mp*(3*t+t) +Mp*(t+t):
> A:=q*3*a*3*t*3/2*a +q*4*a*t*2*a:
> q:=solve(E=A,q);evalf(q);
```

$$\boxed{q := \frac{12}{43}\frac{Mp}{a^2}}$$ decisive

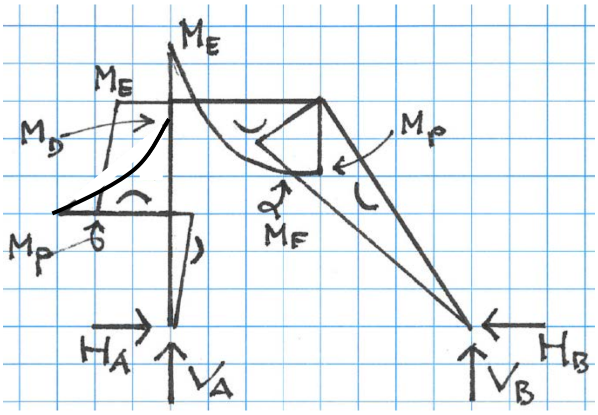
$$\frac{0.2790697674\,Mp}{a^2}$$

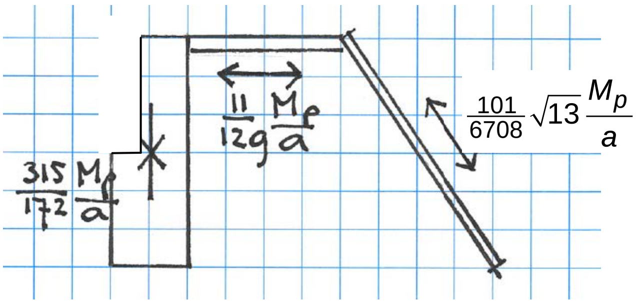
```
> E:=Mp*(t+t) +Mp*(t+t):
> A:=q*3*a*t*3/2*a +q*4*a*t*2*a:
> q:=solve(E=A,q);evalf(q);
```

$$q := \frac{8}{25}\frac{Mp}{a^2}$$

$$\frac{0.3200000000\,Mp}{a^2}$$

Answer to problem 1b






```
> q:=12/43*Mp/a^2:
> eq1:= VA +VB -q*3*a -q*4*a =0:
> eq2:= HA -HB =0:
> eq3:= q*3*a *3/2*a -q*4*a *2*a +VB *8*a=0:
> eq4:= Mp = VB*4*a -HB*6*a:
> opl:=solve({eq1,eq2,eq3,eq4},{VA,VB,HA,HB});
```

$$opl := \left\{HA = -\frac{11}{129}\frac{Mp}{a},\ HB = -\frac{11}{129}\frac{Mp}{a},\ VA = \frac{315}{172}\frac{Mp}{a},\ VB = \frac{21}{172}\frac{Mp}{a}\right\}$$

```
> assign(opl):
> MC:= HB*3*a -VB*8*a +q*4*a*2.*a;
```

$$MC := 1.000000000\ Mp$$

```
> MD:=q*3*a*3/2.*a;
```

$$MD := 1.255813953\ Mp$$

```
> ME:= q*4*a*2*a -VB*8*a +HB*6.*a;
```

$$ME := 0.7441860470\ Mp$$

```
> MF:= VB*5*a -HB*6*a -q*a*1/2.*a;
```

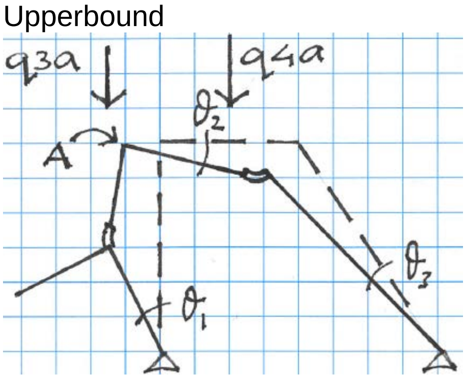
$$MF := 0.9825581396\ Mp$$

```
> l:=sqrt(4^2+6^2):
> N:=VB*6/l+HB*4/l;
```

$$N := \frac{101}{6708}\frac{Mp\sqrt{13}}{a}$$

Answer to problem 1c

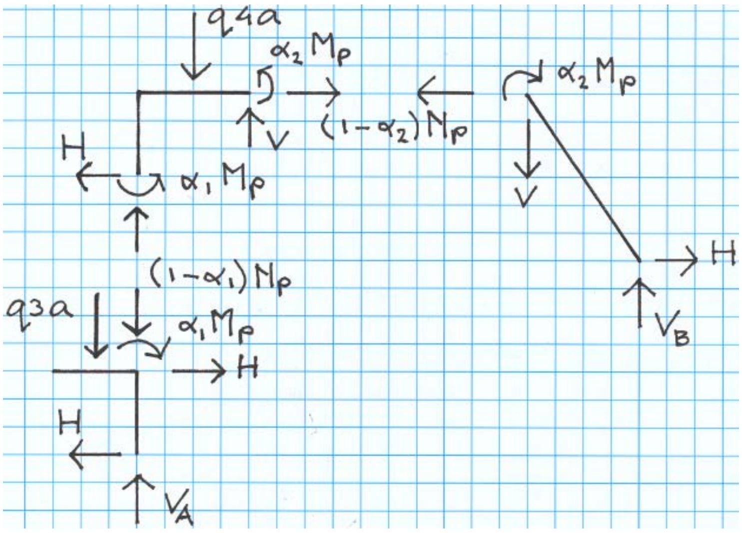
Upperbound



```
> b:=12:
> eq1:= t1*3*a -t2*3*a = t3*6*a +(t2+t3)*a/b:
> eq2:= (t1+t2)*a/b = t3*4*a -t2*4*a:
> E:= Mp*(t1+t2) + Mp*(t2+t3):
> A:= q*3*a *t1*3/2*a +q*4*a *(t3*4*a-t2*2*a):
> solve({eq1,eq2,E=A},{t1,t2,q});
```

$$\left\{q = \frac{6976\,Mp}{26443\,a^2},\ t1 = \frac{5353\,t3}{1801},\ t2 = \frac{1655\,t3}{1801}\right\}$$

Lowerbound




```
> Np:=b*Mp/a:
> eq1:= VA = q*3*a +(1-a1)*Np:
> eq2:= a1*Mp +H*3*a -q*3*a *3/2*a = 0:
> eq3:= (1-a1)*Np +V = q*4*a:
> eq4:= a2*Mp = (1-a1)*Np*4*a -a1*Mp -q*4*a*2*a + H*3*a:
> eq5:= (1-a2)*Np = H:
> eq6:= a2*Mp = VB*4*a +H*6*a:
> eq7:= V = VB:
> solve({eq1,eq2,eq3,eq4,eq5,eq6,eq7},{q,a1,a2,VA,VB,V,H});
```

$$\left\{H = \frac{2340\,Mp}{26443\,a},\ V = \frac{3052\,Mp}{26443\,a},\ VA = \frac{45780\,Mp}{26443\,a},\ VB = \frac{3052\,Mp}{26443\,a},\ a1 = \frac{24372}{26443},\ a2 = \frac{26248}{26443},\ q = \frac{6976\,Mp}{26443\,a^2}\right\}$$

Answer to problem 2a

A, B, D, F

3 or less correct …….0 point
4 correct ……............ 0.3 point
5 correct ……………..0.7 point
6 correct ……………..1.0 point

Answer to problem 2b

```
> E:=mp*4*a*w/(5*a) +2*mp*5*a*w/(4*a) +mp*4*a*w/(5*a) +2*mp*5*a*w/(4*a);
```

$$E := \frac{33}{5}\, mp\, w$$

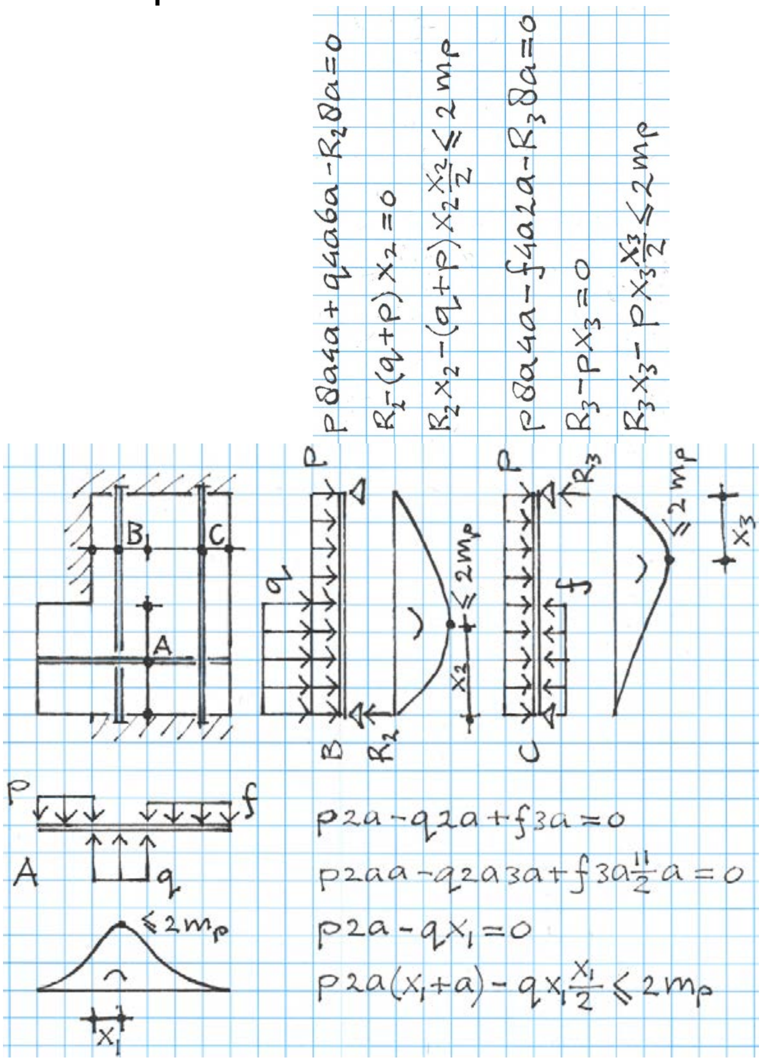
```
> A:=4*(p*1/2*5*a*4*a*w/3) -p*2*a*4*a*w/5;
```

$$A := \frac{176}{15}\, p\, a^2\, w$$

```
> p:=solve(E=A,p);
```

$$p := \frac{9}{16}\, \frac{mp}{a^2}$$

Answer to problem 2c



$$P\,8a\,4a + q\,4a\,6a - R_2\,8a = 0$$

$$R_2 - (q+p)\,x_2 = 0$$

$$R_2 x_2 - (q+p)\,x_2 \frac{x_2}{2} \leq 2m_p$$

$$P\,8a\,4a - f\,4a\,2a - R_3\,8a = 0$$

$$R_3 - p x_3 = 0$$

$$R_3 x_3 - p x_3 \frac{x_3}{2} \leq 2m_p$$

$$p2a - q2a + f3a = 0$$

$$p2aa - q2a3a + f3a\tfrac{11}{2}a = 0$$

$$p2a - qx_1 = 0$$

$$p2a(x_1 + a) - qx_1\frac{x_1}{2} \leq 2m_p$$

The solution to the equations is $p = \dfrac{280}{2209}\dfrac{m_p}{a^2}$.

The latter answer has been computed with the following Maple script.
(Not required for the exam.)

```
> eq1:=p*2*a-q*2*a+f*3*a=0:
> eq2:=p*2*a*a-q*2*a*3*a+f*3*a*11/2*a=0:
> eq3:=p*2*a-q*x1=0:
> eq4:=p*2*a*(x1+a)-q*x1*x1/2=2*mp:
> eq5:=p*8*a*4*a+q*4*a*6*a-R2*8*a=0:
> eq6:=R2-(q+p)*x2=0:
> eq7:=R2*x2-(q+p)*x2*x2/2=2*mp:
> eq8:=p*8*a*4*a-f*4*a*2*a-R3*8*a=0:
> eq9:=R3-p*x3=0:
> eq10:=R3*x3-p*x3*x3/2=2*mp:
> opl:=solve({eq1,eq2,eq3,eq5,eq6,eq7,eq8,eq9},{p,q,f,x1,x2,x3,R2,R3});
```

$$opl := \left\{R2 = \frac{56}{47}\,\frac{mp}{a},\, R3 = \frac{2912}{6627}\,\frac{mp}{a},\, f = \frac{448}{6627}\,\frac{mp}{a^2},\, p = \frac{280}{2209}\,\frac{mp}{a^2},\, q = \frac{504}{2209}\,\frac{mp}{a^2},\, x1 = \frac{10}{9}\,a,\, x2 = \frac{47}{14}\,a,\, x3 = \frac{52}{15}\,a\right\}$$

```
> assign(opl);
> evalf(eq4);
```

$$0.3943463608\, mp = 2.\, mp$$

```
> evalf(eq7);
```

$$2.\, mp = 2.\, mp$$

```
> evalf(eq10);
```

$$0.7616518284\, mp = 2.\, mp$$

Answer to problem 3

a B

b A

c Von Mises, experiments