Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 13 August 2020, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of an elastic team, plastic team or no team.

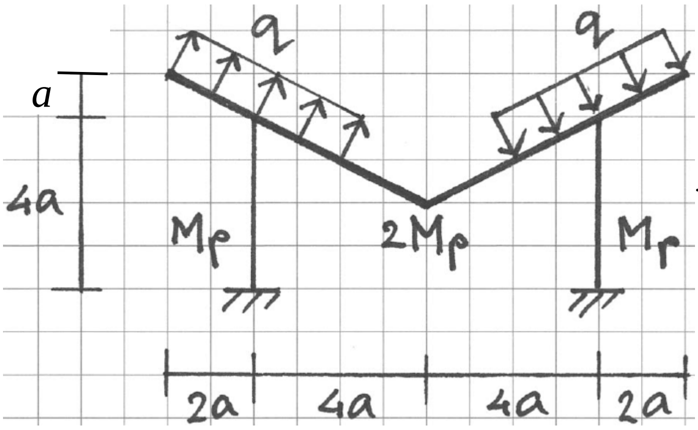


Figure 1. Frame structure

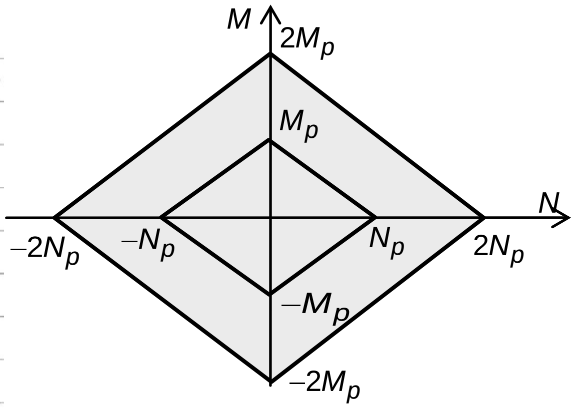


Figure 2. Yield contour

Problem 1

A frame consists of four members (Fig.1). The columns have a strength M_p. The roof members have a strength $2M_p$. All members are rigidly connected. The supports are fixed. The structure is loaded by two evenly distributed line loads q per length of roof member (wind load). The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse (1.5 points).

c Assume β = 12. Choose one of the following problems. (You need not do both.)
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has fixed, hinged and free edges (Fig. 3). It carries an evenly distributed load p [kN/m^2] in the z direction. There is no other load on the plate. The plate is homogeneous and orthotropic. The reinforcement is in the x and y directions. The top reinforcement in the y direction is three times as much as the others.

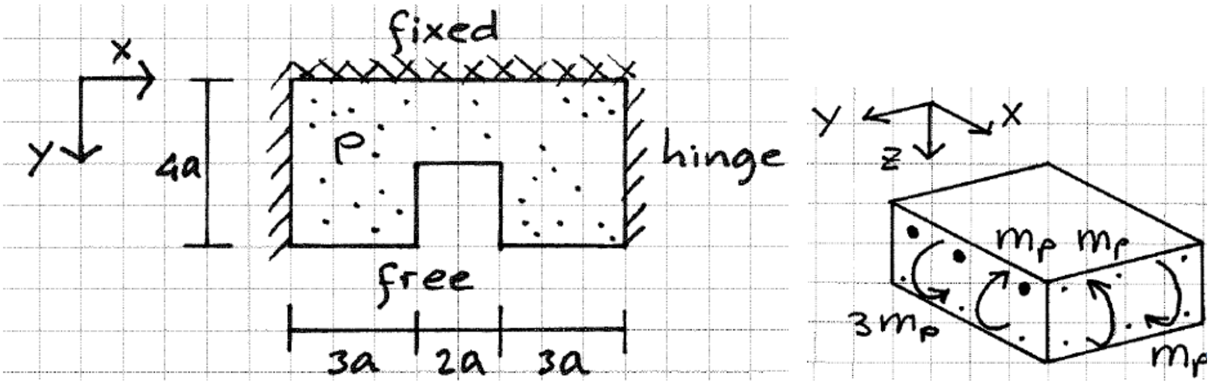


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

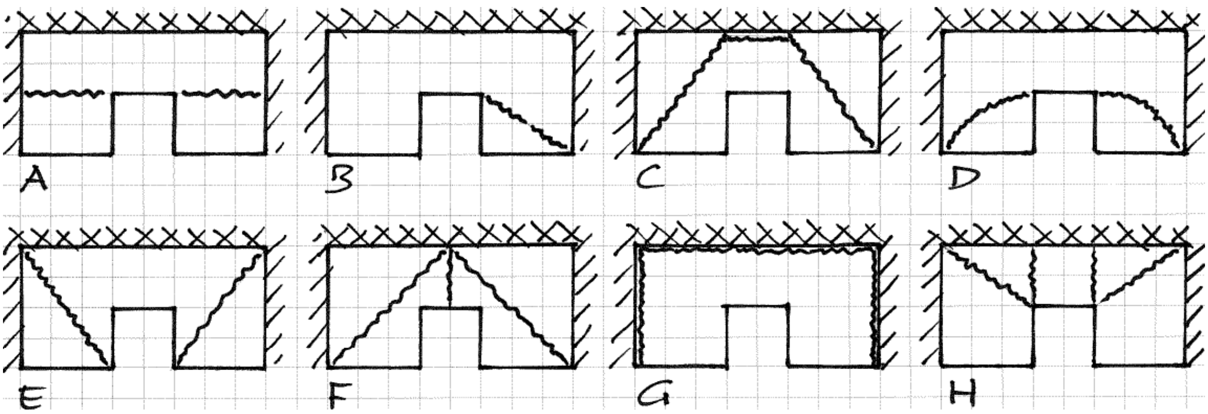


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for p expressed in m_p and a (1.5 point).

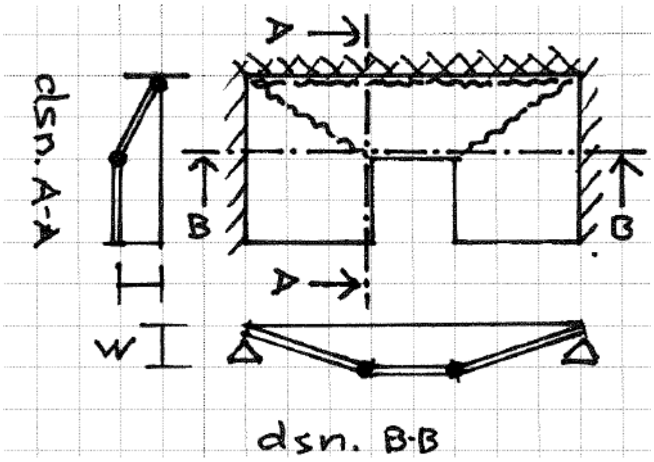


Figure 5. Mechanism of problem 2b

c Determine the largest <u>lower-bound</u> for p using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

Problem 3

a Which yield criterion is used for soil? (0.5 point)

b When do we know that we have found the exact plastic collapse load?
Choose A, B, C or D. (0.5 point)

A …. When we have double checked that no calculation errors were made.
B …. When the lower-bound is the same as the upper-bound.
C …. When a nonlinear finite element analysis gives the same result.
D …. When all possible equilibrium systems have been considered.

c A circular plate is fixed at the edge.
The plate is made of steel.
A perpendicular point load F is applied to middle of the plate.
What is the plastic collapse load?
Choose A, B, C or D. (0.5 point)

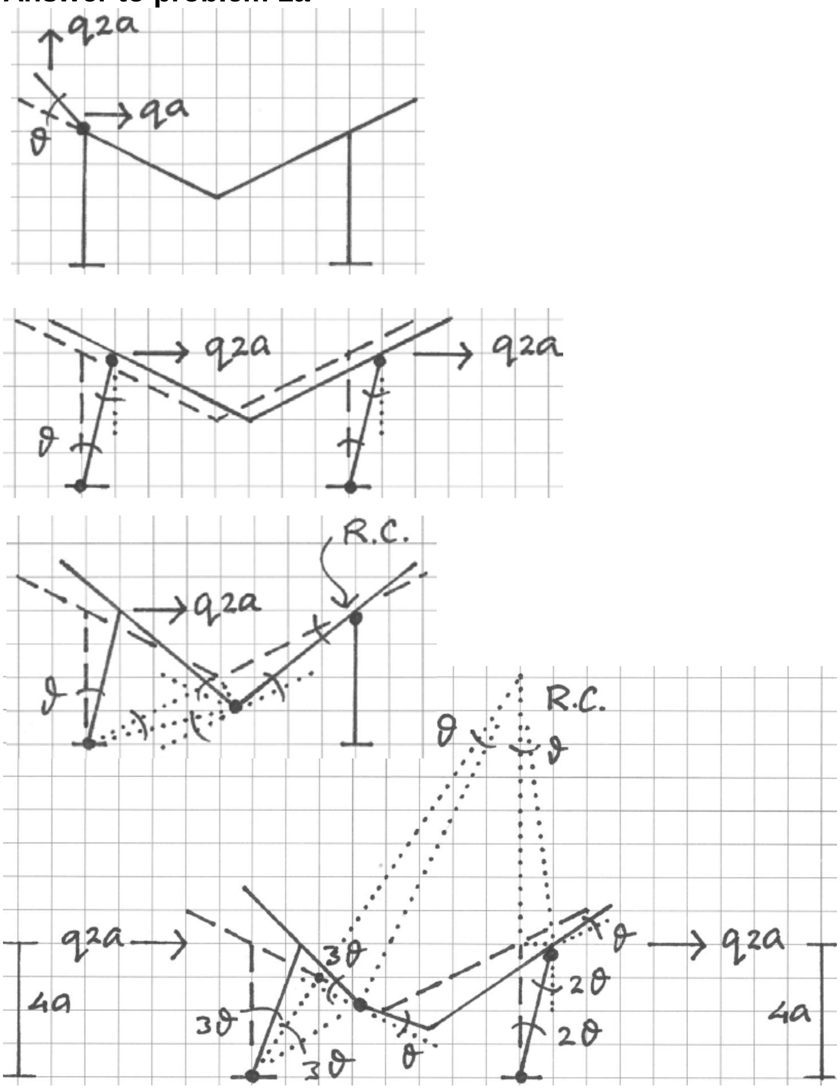
A …. $F = 2\pi m_p$

B …. $F = \frac{4}{\sqrt{3}} \pi m_p$

C …. $F = 4\pi m_p$

D …. F is infinite

Answer to problem 1a



$>$ *restart* :

$>$ $E := 2 \cdot Mp \cdot t$:

$>$ $A := q \cdot 2 \cdot a \cdot t \cdot a + q \cdot a \cdot t \cdot \frac{a}{2}$:

$>$ $q := solve(E = A, q); evalf(q);$

$$q := \frac{4\,Mp}{5\,a^2}$$

$$\frac{0.8000000000\,Mp}{a^2}$$

$>$ *restart* :

$>$ $E := Mp \cdot t + Mp \cdot t + Mp \cdot t + Mp \cdot t$:

$>$ $A := q \cdot 2 \cdot a \cdot t \cdot 4 \cdot a + q \cdot 2 \cdot a \cdot t \cdot 4 \cdot a$:

$>$ $q := solve(E = A, q); evalf(q);$

$$\boxed{q := \frac{Mp}{4\,a^2}}$$ decisive

$$\frac{0.2500000000\,Mp}{a^2}$$

$>$ *restart* :

$>$ $E := Mp \cdot t + 2 \cdot Mp \cdot (t + t) + Mp \cdot t$:

$>$ $A := q \cdot 2 \cdot a \cdot t \cdot 4 \cdot a$:

$>$ $q := solve(E = A, q); evalf(q);$

$$q := \frac{3\,Mp}{4\,a^2}$$

$$\frac{0.7500000000\,Mp}{a^2}$$

$>$ *restart* :

$>$ $E := Mp \cdot 3 \cdot t + 2 \cdot Mp \cdot (3 \cdot t + t) + Mp \cdot (t + 2 \cdot t) + Mp \cdot 2 \cdot t$:

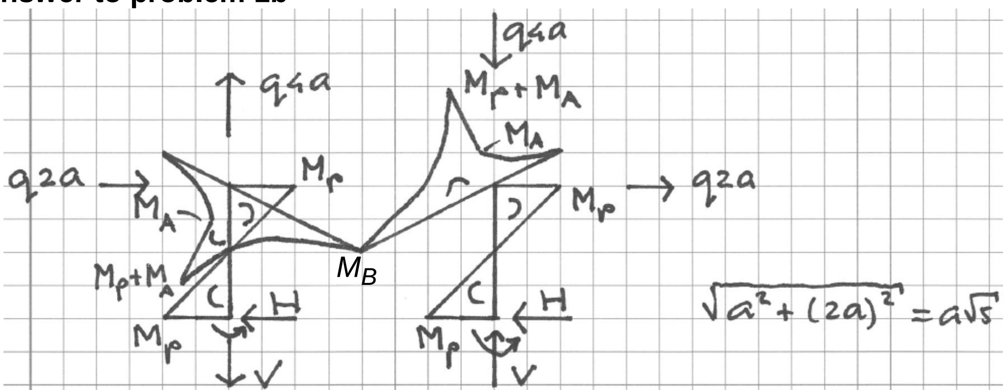
$>$ $A := q \cdot 2 \cdot a \cdot 3 \cdot t \cdot 4 \cdot a + q \cdot 2 \cdot a \cdot 2 \cdot t \cdot 4 \cdot a$:

$>$ $q := solve(E = A, q); evalf(q);$

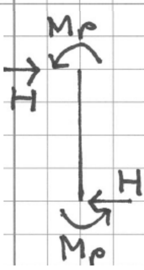
$$q := \frac{2\,Mp}{5\,a^2}$$

$$\frac{0.4000000000\,Mp}{a^2}$$

Answer to problem 1b



$$\sqrt{a^2 + (2a)^2} = a\sqrt{5}$$

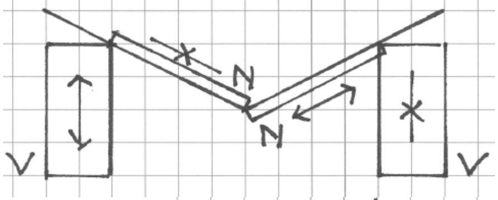


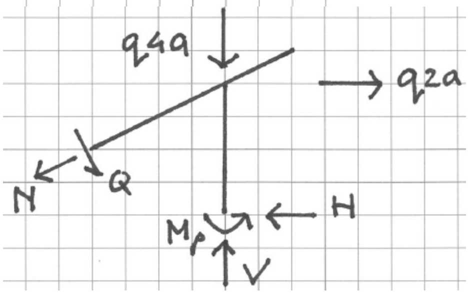
$$M_A = qa\sqrt{5}\,\tfrac{1}{2}a\sqrt{5} = \tfrac{5}{2}qa^2 = \tfrac{5}{8}M_p$$

$$H4a = 2M_p \Rightarrow H = \tfrac{1}{2}\frac{M_p}{a}$$

$$M_p = q2a4a + q4a8a + q2a4a - M_p - V8a \Rightarrow V = \frac{5}{4}\frac{M_p}{a}$$

$$M_B = q4a4a + q2a2a - V4a + H2a - M_p = 0$$

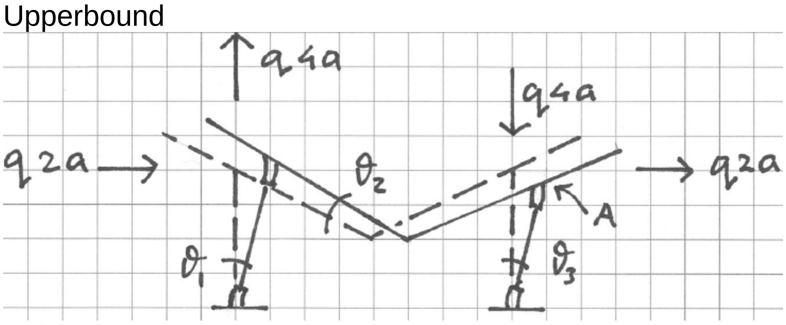




$$\left.\begin{aligned} Q\frac{1}{\sqrt{5}} - N\frac{2}{\sqrt{5}} &= H - q2a \\ Q\frac{2}{\sqrt{5}} + N\frac{1}{\sqrt{5}} &= V - q4a \end{aligned}\right\} \quad \begin{aligned} Q &= \frac{\sqrt{5}}{10}\frac{M_p}{a} \\ N &= \frac{\sqrt{5}}{20}\frac{M_p}{a} \end{aligned}$$

Answer to problem 1c

Upperbound



> $b := 12:$

> $eq1 := t1\cdot 4\cdot a = t3\cdot 4\cdot a:$ # *horizontal displacement of A*

> $eq2 := t1\cdot\frac{a}{b} + (t1 - t2)\cdot\frac{a}{b} - t2\cdot 8\cdot a = -t3\cdot\frac{a}{b} - (t3 - t2)\cdot\frac{a}{b}:$ # *vertical displacement of A*

> $E := Mp\cdot t1 + Mp\cdot(t1 - t2) + Mp\cdot(t3 - t2) + Mp\cdot t3:$

> $A := q\cdot 2\cdot a\cdot t1\cdot 4\cdot a + q\cdot 4\cdot a\cdot\left(t1\cdot\frac{a}{b} + (t1 - t2)\cdot\frac{a}{b}\right) + q\cdot 4\cdot a\cdot\left(t3\cdot\frac{a}{b} + (t3 - t2)\cdot\frac{a}{b}\right) + q\cdot 2\cdot a\cdot t3\cdot 4\cdot a:$

> $solve(\{eq1, eq2, E = A\}, \{q, t2, t3\});$

$$\left\{q = \frac{12\,Mp}{53\,a^2},\ t2 = \frac{2\,t1}{49},\ t3 = t1\right\}$$

Lowerbound

> $b := 12 : Np := b\cdot\frac{Mp}{a}:$

> $eq1 := a1\cdot Mp + a1\cdot Mp = H1\cdot 4\cdot a:$

> $eq2 := q\cdot 2\cdot a + q\cdot 2\cdot a = H1 + H2:$

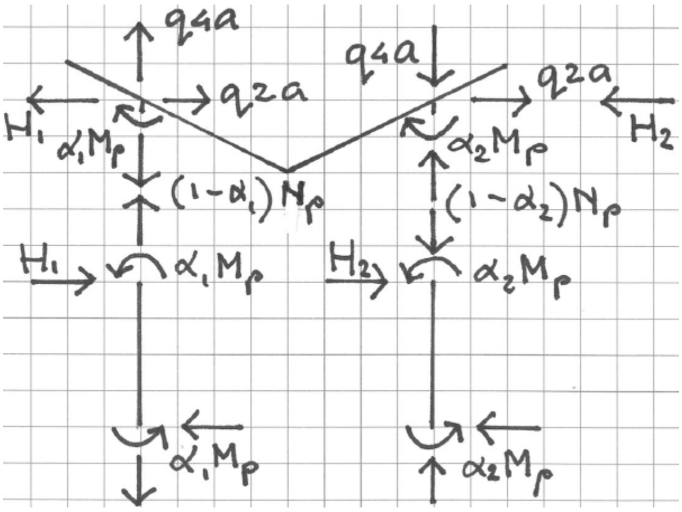
> $eq3 := q\cdot 4\cdot a - q\cdot 4\cdot a = (1 - a1)\cdot Np - (1 - a2)\cdot Np:$

> $eq4 := a1\cdot Mp + a2\cdot Mp = ((1 - a2)\cdot Np - q\cdot 4\cdot a)\cdot 8\cdot a:$

> $eq5 := a2\cdot Mp + a2\cdot Mp = H2\cdot 4\cdot a:$

> $solve(\{eq1, eq2, eq3, eq4, eq5\}, \{q, a1, a2, H1, H2\});$

$$\left\{H1 = \frac{24\,Mp}{53\,a},\ H2 = \frac{24\,Mp}{53\,a},\ a1 = \frac{48}{53},\ a2 = \frac{48}{53},\ q = \frac{12\,Mp}{53\,a^2}\right\}$$



$$> \frac{12}{53.};$$

$$0.2264150944$$

Answer to problem 2a
B, F

4 or less correct ……. 0.0 point
5 correct …………….. 0.2 point
6 correct …………….. 0.5 point
7 correct …………….. 0.8 point
8 correct ……………..1.0 point

Answer to problem 2b

$$> E := 3\cdot mp\cdot 8\cdot a\cdot \frac{w}{2\cdot a} + 2\cdot\left(mp\cdot 3\cdot a\cdot \frac{w}{2\cdot a}\right) + 2\cdot\left(mp\cdot 2\,a\cdot \frac{w}{3\cdot a}\right);$$

$$E := \frac{49\,mp\,w}{3}$$

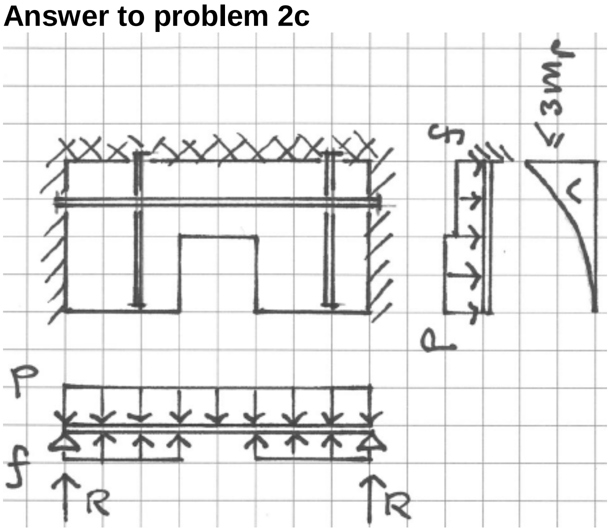
$$> A := 2\cdot\left(p\cdot 2\cdot a\cdot 3\cdot a\cdot \frac{w}{2}\right) + 4\cdot\left(p\cdot \frac{2\cdot a\cdot 3\cdot a}{2}\cdot \frac{w}{3}\right) + p\cdot 2\cdot a\cdot 2\cdot a\cdot \frac{w}{2};$$

$$A := 12\,p\,a^2\,w$$

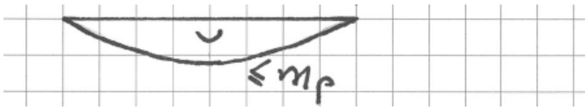
$$> p := solve(E = A, p);$$

$$p := \frac{49\,mp}{36\,a^2}$$

Answer to problem 2c



$$M = p\,2a\,3a + f\,2a\,a \le 3m_p$$



$$p\,8a - f\,3a - f\,3a - R - R = 0$$

$$m = R\,4a + f\,3a\,\tfrac{5}{2}a - p\,4a\,2a \leq m_p$$

The solution to the equations is

$$p = \frac{31}{86}\frac{m_p}{a^2} \qquad f = \frac{18}{43}\frac{m_p}{a^2} \qquad R = \frac{8}{43}\frac{m_p}{a} \qquad \frac{31}{86} = 0{,}36$$

Answer to problem 3

a Mohr-Coulomb

b B

c B …… (A is Tresca, B is Von Mises, C is Rankine, D is nonsense)