Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 22 January 2020, 9:00 – 12:00 hours

Write your name and study number at the top of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

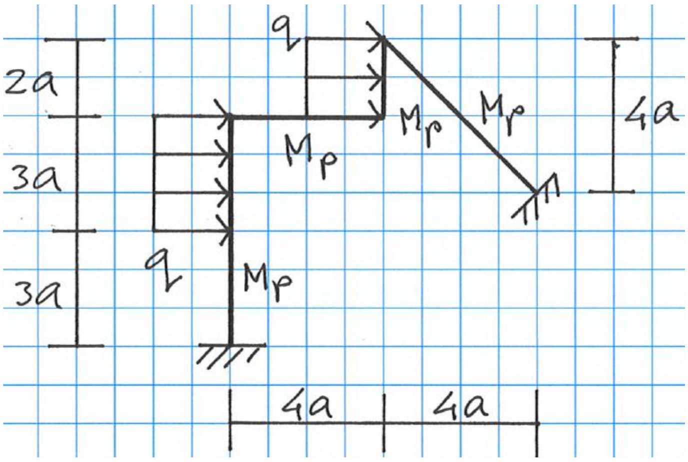


Figure 1. Frame structure

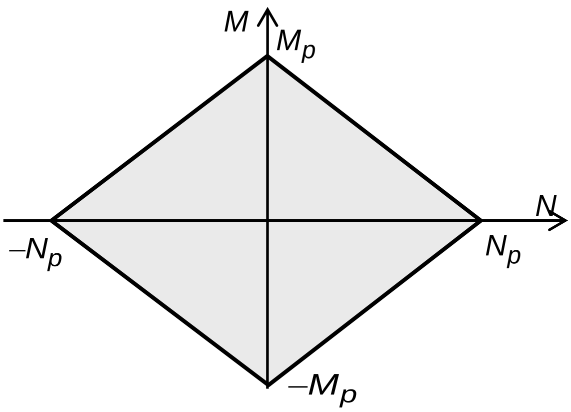


Figure 2. Yield contour

Problem 1

A frame consists of four members(Fig.1). All members have a strength M_p. The members are rigidly connected. The supports are fixed. The structure is loaded by two evenly distributed line loads q. The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

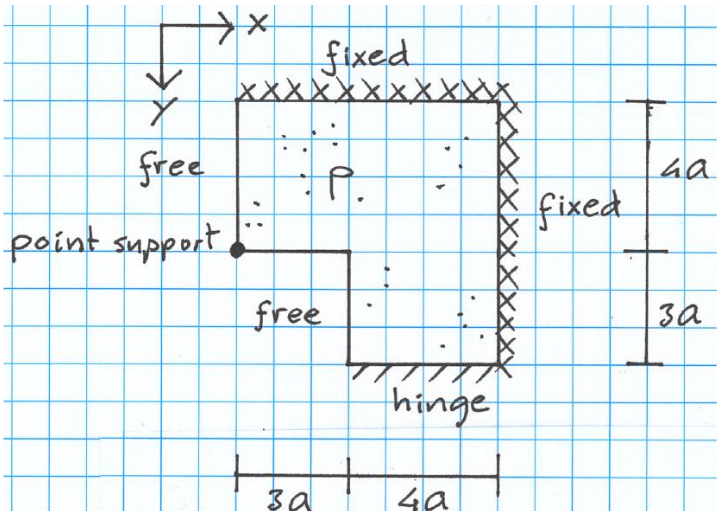
a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse (1.5 points).

c Assume β = 18. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has fixed, hinged and free edges (Fig. 3). It carries an evenly distributed load p [kN/m^2]. There is no other load on the plate. The plate is homogeneous and orthotropic.



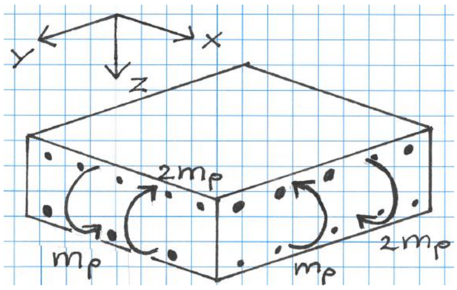


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

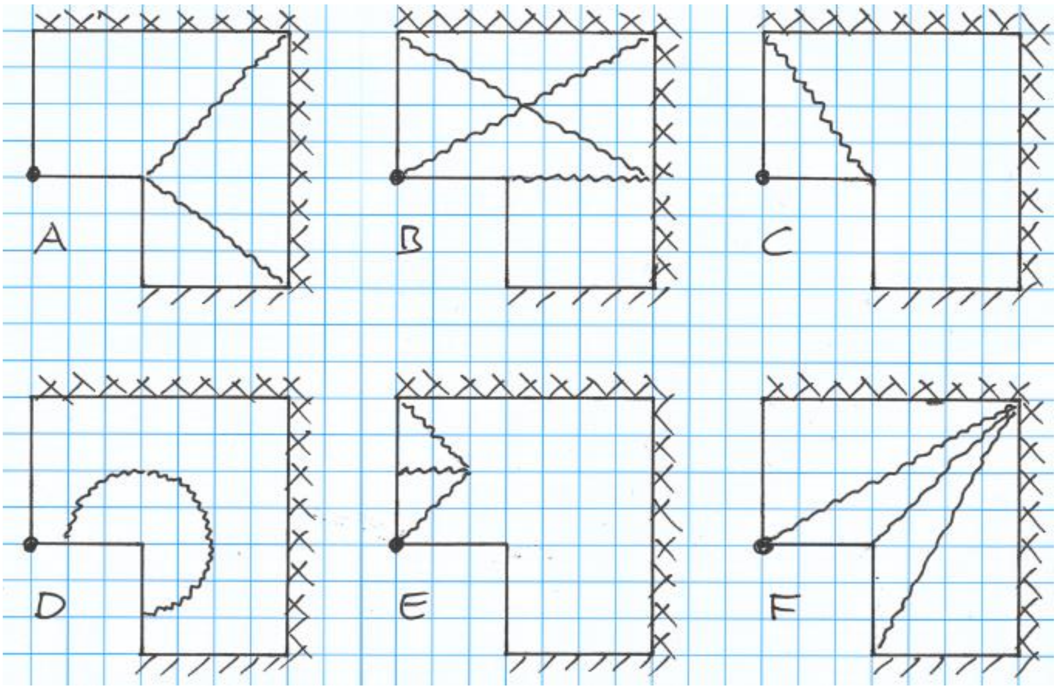


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for p expressed in m_p and a (1.5 point).

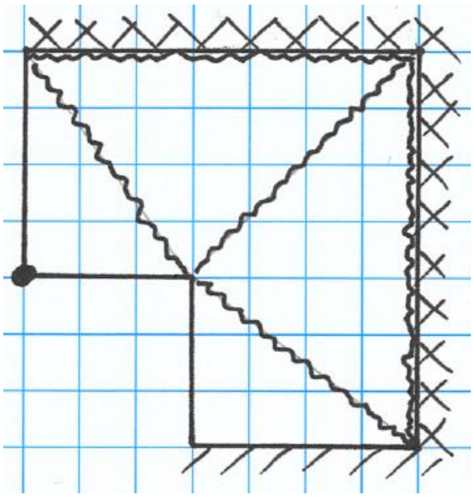


Figure 5. Mechanism of problem 2b

c Determine the largest <u>lower-bound</u> for p using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

Problem 3

a Which of the following words does not belong in this list? Choose A, B, C or D (0.5 point).

A Yield contour
B Limit state function
C Response surface
D Moment-curvature diagram

b How do we know that the limit load of a plate is exact?
Choose A, B, C, or D (0.5 point).

A It agrees with the result of a non-linear finite element analysis,
B It is the largest lower bound,
C It is the largest upper bound,
D The upper bound and the lower bound are the same.

c Consider a structure. The degree of indeterminancy is n. A mechanism has less than n+1 plastic hinges. How do we call this mechanism? (0.5 point)

Answer to problem 1a

> restart :

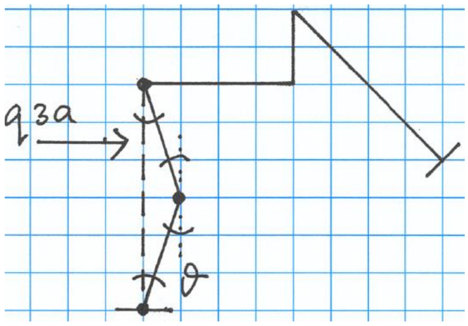
> $E := Mp\cdot t + Mp\cdot(t + t) + Mp\cdot t:$

> $A := q\cdot 3\cdot a\cdot \frac{3}{2}\cdot a\cdot t:$

> $q := solve(E = A, q);\ evalf(q);$

$$q := \frac{8}{9}\,\frac{Mp}{a^2}$$

$$\frac{0.8888888889\,Mp}{a^2}$$



> restart :

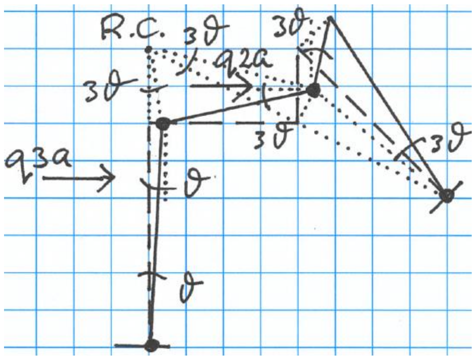
> $E := Mp\cdot t + Mp\cdot(t + 3\cdot t) + Mp\cdot(3\cdot t + 3\cdot t) + Mp\cdot 3\cdot t:$

> $A := q\cdot 3\,a\cdot \frac{9}{2}\cdot a\cdot t + q\cdot 2\cdot a\cdot 3\cdot a\cdot 3\cdot t:$

> $q := solve(E = A, q);\ evalf(q);$

$$q := \frac{4\,Mp}{9\,a^2}$$

$$\frac{0.4444444444\,Mp}{a^2}$$



> restart :

> $E := Mp\cdot t + Mp\cdot(t + t) + Mp\cdot(t + t) + Mp\cdot t:$

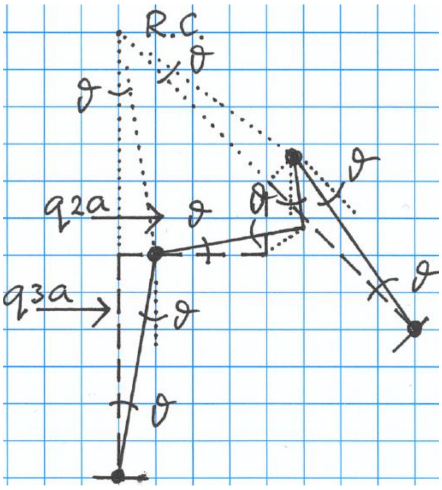
>

> $A := q\cdot 3\cdot a\cdot \frac{9}{2} a\cdot t + q\cdot 2\cdot a\cdot 5\cdot a\cdot t:$

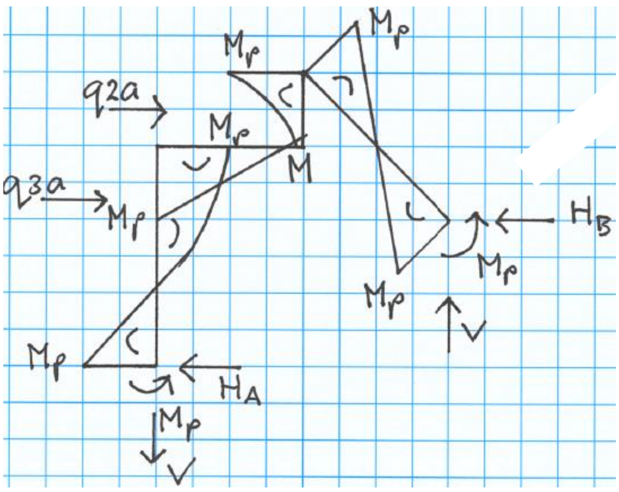
> $q := solve(E = A, q);\ evalf(q);$

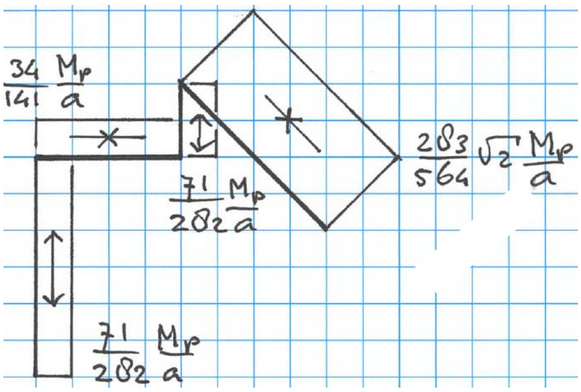
$$\boxed{q := \frac{12}{47}\,\frac{Mp}{a^2}}$$ decisive

$$\frac{0.2553191489\,Mp}{a^2}$$



Answer to problem 1b





$> q := \frac{12}{47} \frac{Mp}{a^2}:$

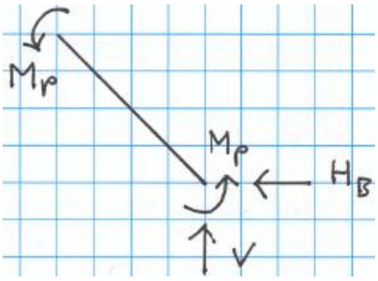
$> eq1 := q \cdot 3 \cdot a + q \cdot 2 \cdot a = HA + HB:$

$> eq2 := Mp = q \cdot 3 \cdot a \cdot \frac{9}{2} \cdot a + q \cdot 2 \cdot a \cdot 7 \cdot a - Mp - V \cdot 8 \cdot a - HB \cdot 4 \cdot a:$

$> eq3 := Mp = HB \cdot 4 \cdot a - V \cdot 4 \cdot a - Mp:$

$> opl := solve(\{eq1, eq2, eq3\}, \{HA, HB, V\}); \quad assign(opl):$

$$opl := \left\{HA = \frac{74}{141} \frac{Mp}{a}, HB = \frac{106}{141} \frac{Mp}{a}, V = \frac{71}{282} \frac{Mp}{a}\right\}$$



$> M := Mp + V \cdot 4 \cdot a - HB \cdot 2 \cdot a - q \cdot 2 \cdot a \cdot a;$

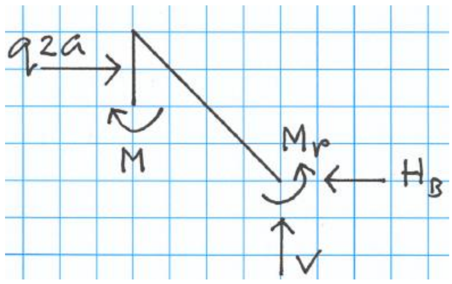
$$M := -\frac{1}{141} Mp$$

$$-0.007092198582\ Mp$$

$> N1 := V : N2 := q \cdot 3 \cdot a - HA; N3 := V : N4 := \frac{V}{\mathrm{sqrt}(2)} + \frac{HB}{\mathrm{sqrt}(2)};$

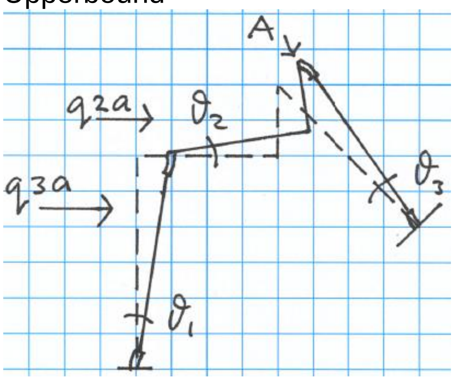
$$N2 := \frac{34}{141} \frac{Mp}{a}$$

$$N4 := \frac{283}{564} \frac{Mp\sqrt{2}}{a}$$



Answer to problem 1c

Upperbound



$> b := 18.:$

$> eq1 := t1 \cdot 6 \cdot a - t2 \cdot 2 \cdot a \ = \ t3 \cdot 4 \cdot a + t3 \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} + (t2 + t3) \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)}:$

$> eq2 := t1 \cdot \frac{a}{b} + (t1 + t2) \cdot \frac{a}{b} + t2 \cdot 4 \cdot a \ = \ t3 \cdot 4 \cdot a - t3 \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} - (t2 + t3) \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)}:$

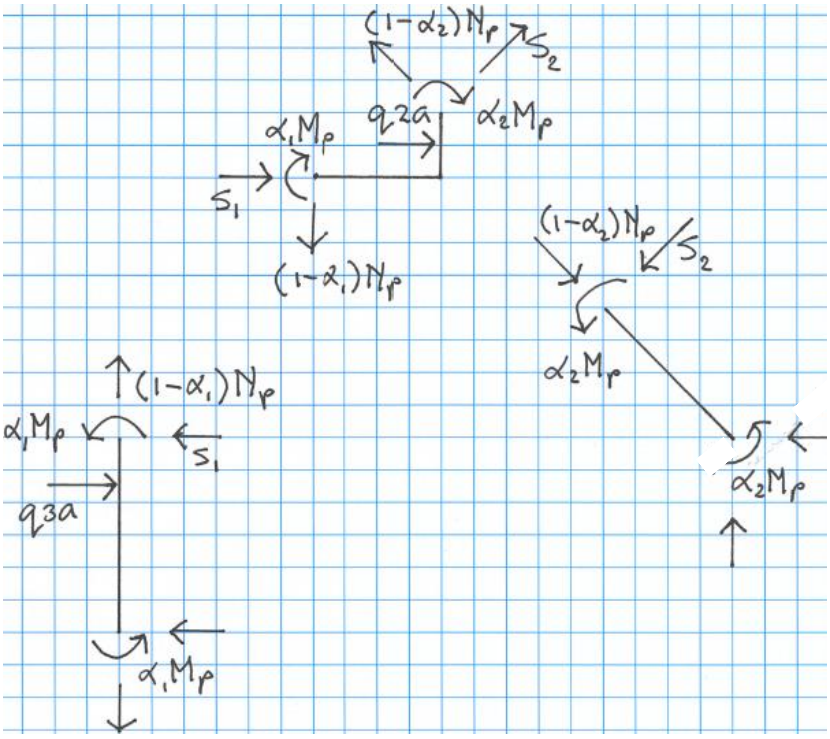
$> E := Mp \cdot t1 + Mp \cdot (t1 + t2) + Mp \cdot (t2 + t3) + Mp \cdot t3:$

$> A := q \cdot 3 \cdot a \cdot \frac{9}{2} \cdot a \cdot t1 + q \cdot 2 \cdot a \cdot (t1 \cdot 6 \cdot a - t2 \cdot a):$

$> solve(\{eq1, eq2, E = A\}, \{t1, t3, q\});$

$$\left\{q = \frac{0.2486740044\ Mp}{a^2}, t1 = 1.070315955\ t2, t3 = 1.074546910\ t2\right\}$$

Lowerbound



$> b := 18. : Np := b \cdot \frac{Mp}{a} :$

$> eq1 := a1 \cdot Mp = q \cdot 3 \cdot a \cdot \frac{9}{2} \cdot a - a1 \cdot Mp - S1 \cdot 6 \cdot a :$

$> eq2 := S1 + q \cdot 2 \cdot a + \frac{S2}{\mathrm{sqrt}(2)} - \frac{(1 - a2) \cdot Np}{\mathrm{sqrt}(2)} = 0 :$

$> eq3 := (1 - a1) \cdot Np = \frac{S2}{\mathrm{sqrt}(2)} + \frac{(1 - a2) \cdot Np}{\mathrm{sqrt}(2)} :$

$> eq4 := a2 \cdot Mp = q \cdot 2 \cdot a \cdot a + S1 \cdot 2 \cdot a + (1 - a1) \cdot Np \cdot 4 \cdot a - a1 \cdot Mp :$

$> eq5 := -a2 \cdot Mp = a2 \cdot Mp + S2 \cdot 4 \cdot a \cdot \mathrm{sqrt}(2) :$

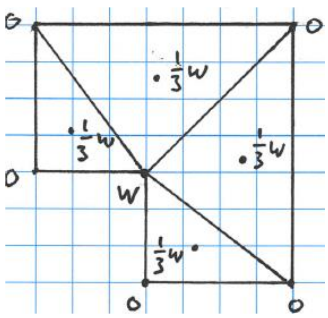
$> solve(\{eq1, eq2, eq3, eq4, eq5\}, \{S1, S2, a1, a2, q\});$

$$\left\{S1 = \frac{0.2307622225\ Mp}{a}, S2 = -\frac{0.3400064641\ Mp}{a}, a1 = 0.9862628623, a2 = 0.9616835056, q = \frac{0.2486740044\ Mp}{a^2}\right\}$$

Answer to problem 2a
E, F

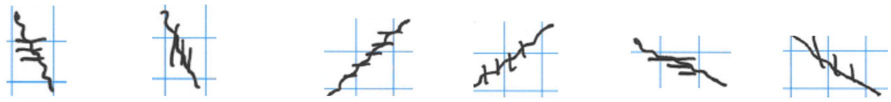
3 or less correct ……. 0 point
4 correct ……............ 0.3 point
5 correct ……………..0.7 point
6 correct ……………..1.0 point

Answer to problem 2b



$> E := p \cdot \frac{3 \cdot a \cdot 4 \cdot a}{2} \cdot \frac{w}{3} + p \cdot \frac{7 \cdot a \cdot 4 \cdot a}{2} \cdot \frac{w}{3} + p \cdot \frac{7 \cdot a \cdot 4 \cdot a}{2} \cdot \frac{w}{3} + p \cdot \frac{3 \cdot a \cdot 4 \cdot a}{2} \cdot \frac{w}{3};$

$$E := \frac{40}{3}\, p\, a^2\, w$$



$> A := mp\cdot 4\cdot a\cdot\frac{w}{3\cdot a}+2\cdot mp\cdot 3\cdot a\cdot\frac{w}{4\cdot a}+mp\cdot 4\cdot a\cdot\frac{w}{4\cdot a}+2\cdot mp\cdot 4\cdot a\cdot\frac{w}{4\cdot a}+mp\cdot 3\cdot a\cdot\frac{w}{4\cdot a}+2\cdot mp\cdot 4\cdot a\cdot\frac{w}{3\cdot a}+2\cdot mp\cdot 7\cdot a\cdot\frac{w}{4\cdot a}+mp\cdot 7\cdot a\cdot\frac{w}{4\cdot a};$

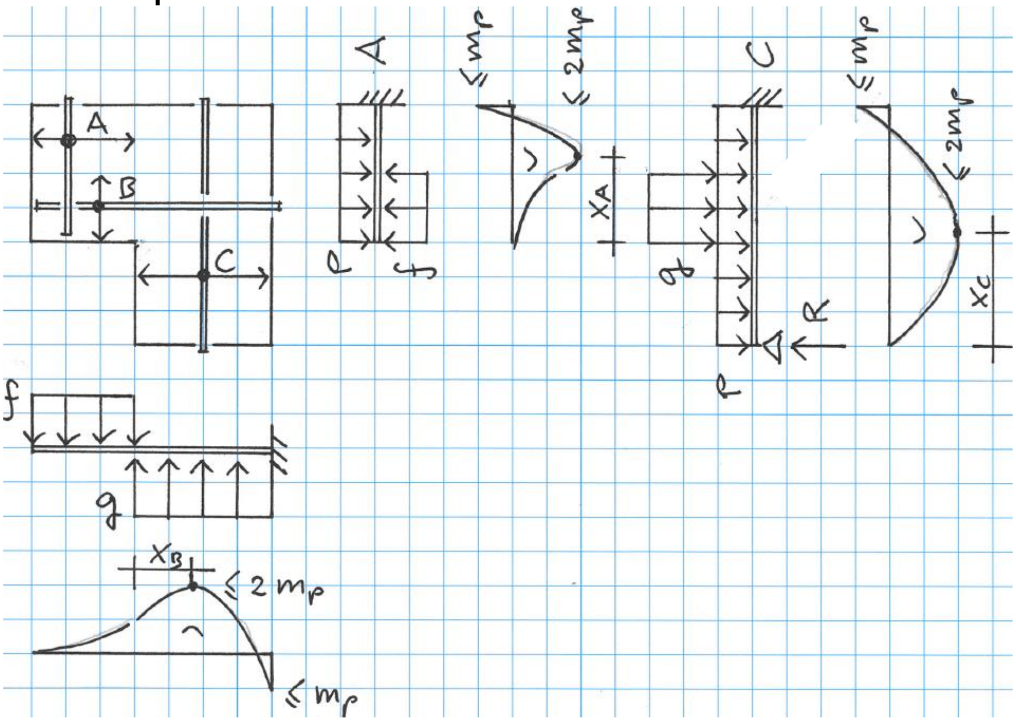
$$A := \frac{29}{2}\,mp\,w$$

$> p := solve(E = A, p);\;\; evalf(p);$

$$p := \frac{87}{80}\,\frac{mp}{a^2}$$

$$\frac{1.087500000\,mp}{a^2}$$

Answer to problem 2c



We cannot use the point support because torsion is neglected.

$> eq1 := f\cdot 3\cdot a = g\cdot xB:$

$> eq2 := f\cdot 3\cdot a\cdot\left(\frac{3}{2}\cdot a+xB\right)-g\cdot xB\cdot\frac{xB}{2}=2\cdot mp:$

$> eq3 := g\cdot 4\cdot a\cdot 2\cdot a-f\cdot 3\cdot a\cdot\frac{11}{2}\cdot a=mp:$

$> eq4 := f\cdot 2\,a=p\cdot xA:$

$> eq5 := f\cdot 2\cdot a\cdot(xA-a)-p\cdot xA\cdot\frac{xA}{2}=2\cdot mp:$

$> eq6 := p\cdot 4\cdot a\cdot 2\cdot a-f\cdot 2\cdot a\cdot 3\cdot a=mp:$

$> eq7 := R=p\cdot xC+g\cdot(xC-3\cdot a):$

$> eq8 := R\cdot xC-p\cdot xC\cdot\frac{xC}{2}-g\cdot(xC-3\cdot a)\cdot\frac{(xC-3\cdot a)}{2}=2\cdot mp:$

$> eq9 := p\cdot 7\cdot a\cdot\frac{7}{2}\cdot a+g\cdot 2\cdot a\cdot 3\cdot a-R\cdot 7\cdot a=mp:$

> $opl := solve(\{eq1, eq2, eq4, eq6, eq7, eq8, eq9\}, \{xA, xB, xC, R, f, g, p\}) : assign(opl);$

> $evalf(p);$

$$\frac{0.2956414615\ mp}{a^2}$$

> $evalf(eq3);$

$$-1.844997579\ mp = mp$$

> $evalf(eq5);$

$$-0.1048478393\ mp = 2.\ mp$$

Answer to problem 3

a D

b D

c Partial mechanism