Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Wednesday 27 May 2020, 13:30 – 16:30 hours

Write your name and study number at the top of your work.

Also write whether you were a member of an elastic team, plastic team or no team.

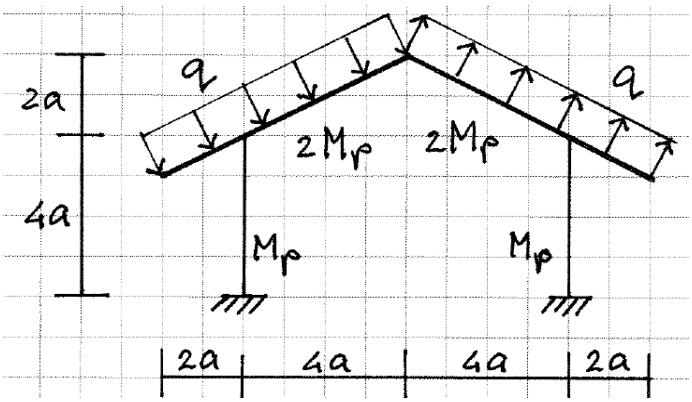


Figure 1. Frame structure

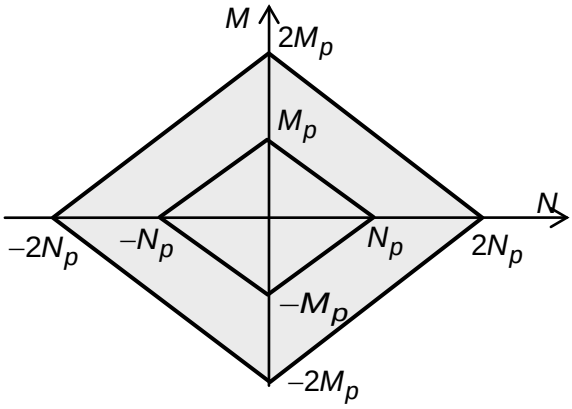


Figure 2. Yield contour

Problem 1

A frame consists of four members (Fig.1). The columns have a strength M_p. The roof members have a strength $2M_p$. All members are rigidly connected. The supports are fixed. The structure is loaded by two evenly distributed line loads q per length of roof member (wind load). The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse (1.5 points).

c Assume $\beta = 14$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has hinged and free edges (Fig. 3). It carries an evenly distributed load p [kN/m^2]. There is no other load on the plate. The plate is homogeneous and orthotropic. The reinforcement is in the x and y directions. The top reinforcement in the y direction is three times as much as the others.

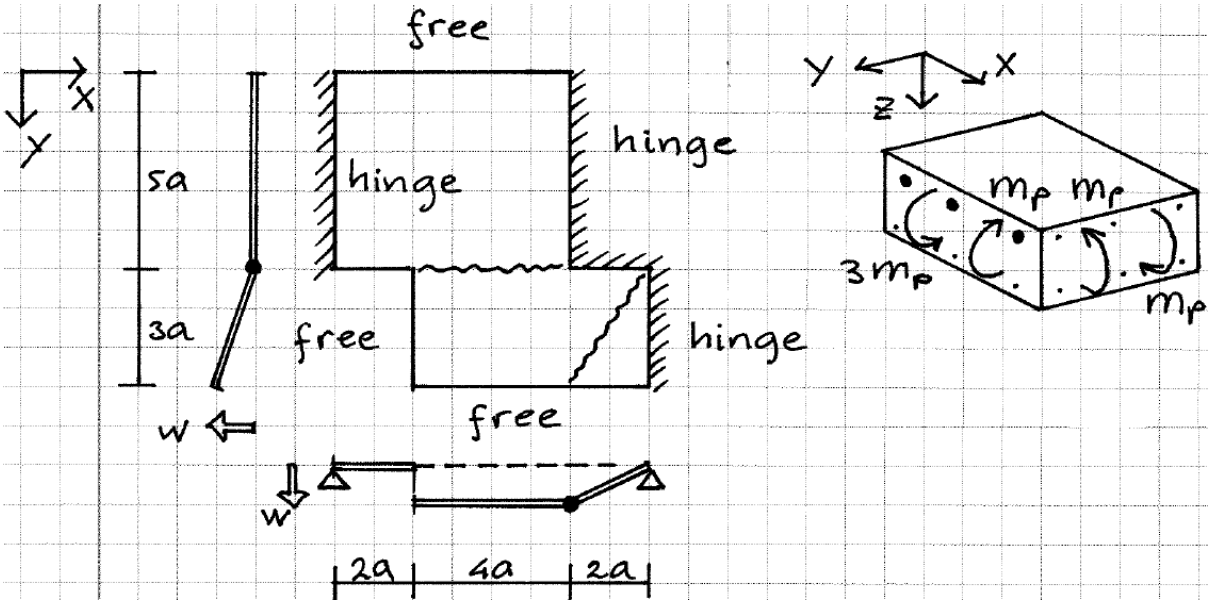


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

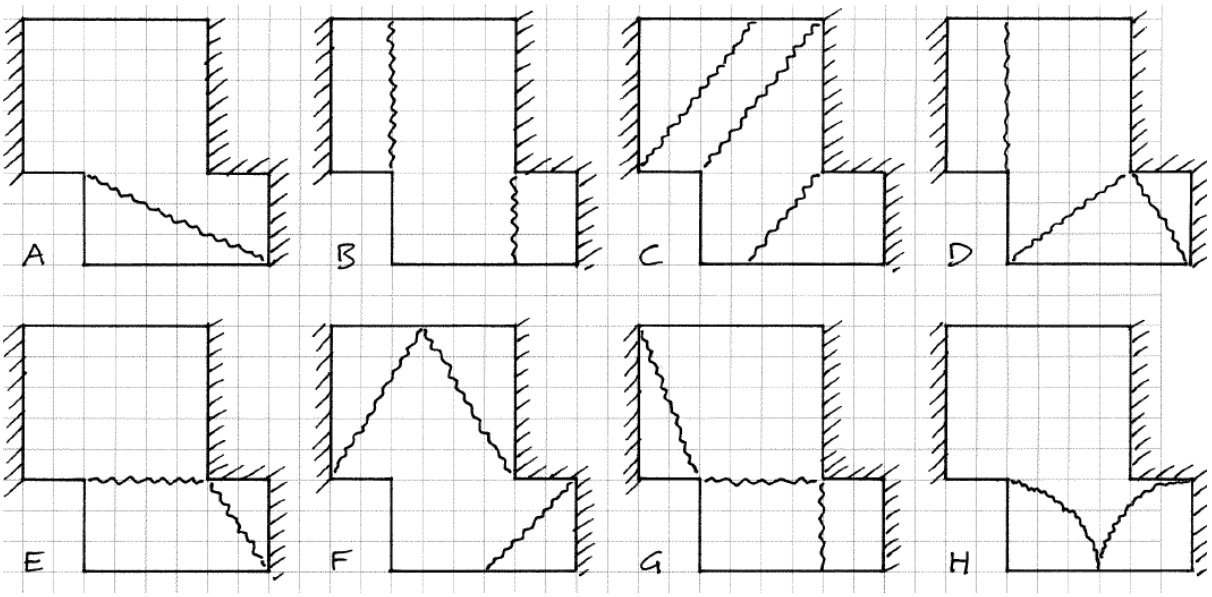


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an upper bound for *p* expressed in m_p and *a* (1.5 point).

c Determine the largest lower-bound for *p* using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

Problem 3

a Consider the following yield criterion.

$\sigma_1 - \sigma_3 < \sigma_v$

where

$\sigma_3 \le \sigma_2 \le \sigma_1$ ..... are the ordered principal stresses and
σ_v ..... is the yield stress.

What is the name of this yield criterion? (0.5 point)

b Sort the following words in two groups? (0.5 point)
equilibrium, virtual work equation, upperbound, safe, mechanism, lowerbound

c Consider a circular plate that is simply supported.

The plate fails at a distributed load $q = 24\frac{m_p}{a^2}$ ..... (reader Plates, p. 40)

The plate fails at a point load $F = 2\pi m_p$ ..... (reader Plates, p. 43)

Suppose that the loads are applied at the same time. At what load does the plate fail?

Choose A, B, C or D. (0.5 point)

A .... $q + F = 24\frac{m_p}{a^2} + 2\pi m_p$

B .... $q + \frac{F}{\pi a^2} = 24\frac{m_p}{a^2}$

C .... $qa^2 + F = (24 + 2\pi)m_p$

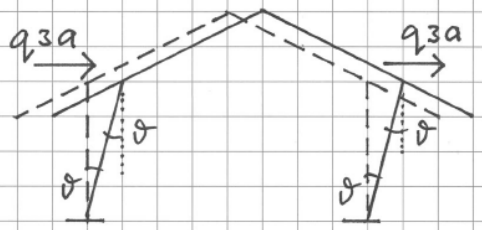
D .... $\frac{qa^2}{24} + \frac{F}{2\pi} = m_p$

Answer to problem 1a

$$E = 2M_p\vartheta$$

$$A = q2a\,a\vartheta + qa\,\frac{a}{2}\vartheta$$

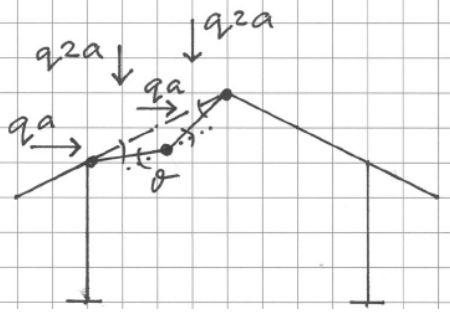
$$E = A \Rightarrow q = \frac{4}{5}\frac{M_p}{a^2}$$



$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta$$

$$A = q3a\,\vartheta\,4a\,2$$

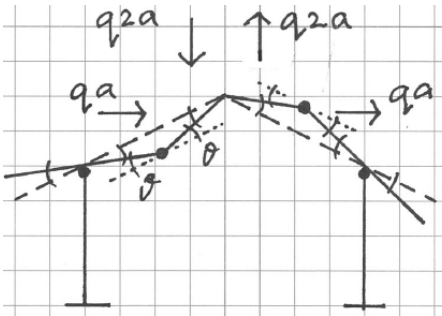
$$E = A \Rightarrow \boxed{q = \frac{1}{6}\frac{M_p}{a^2}}$$ decisive



$$E = 2M_p\vartheta + 2M_p(\vartheta+\vartheta) + 2M_p\vartheta$$

$$A = 2\left[q2a\,\vartheta a + qa\,\vartheta\,\frac{a}{2}\right]$$

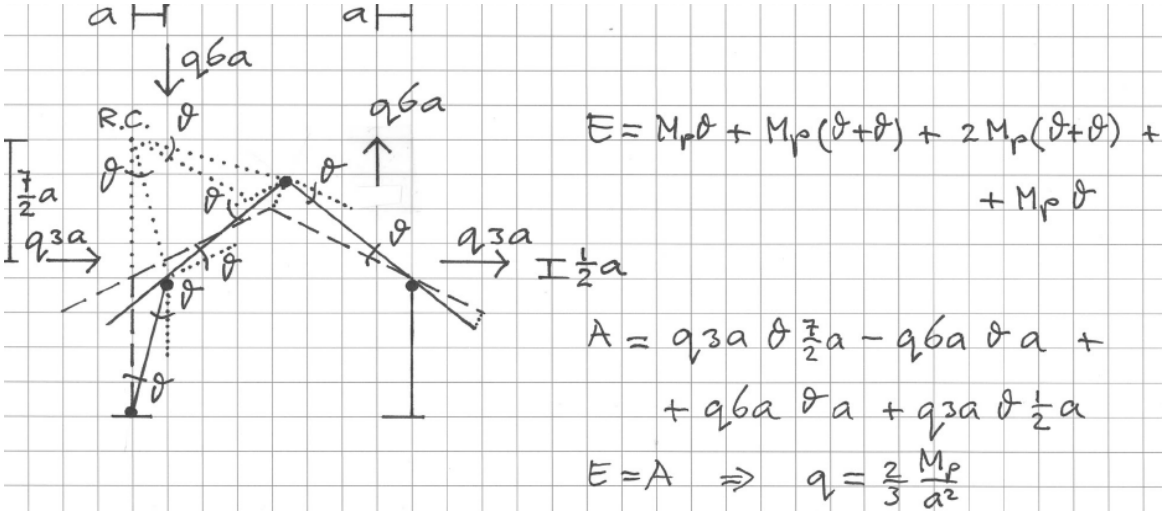
$$E = A \Rightarrow q = \frac{8}{5}\frac{M_p}{a^2}$$



$$E = M_p\vartheta + 2M_p(\vartheta+\vartheta) + 2M_p(\vartheta+\vartheta) + M_p\vartheta$$

$$A = qa\,\vartheta\,\tfrac{1}{2}a + q2a\,\vartheta a + q2a\,\vartheta a + qa\,\vartheta\,\tfrac{1}{2}a$$

$$E = A \Rightarrow q = 2\frac{M_p}{a^2}$$

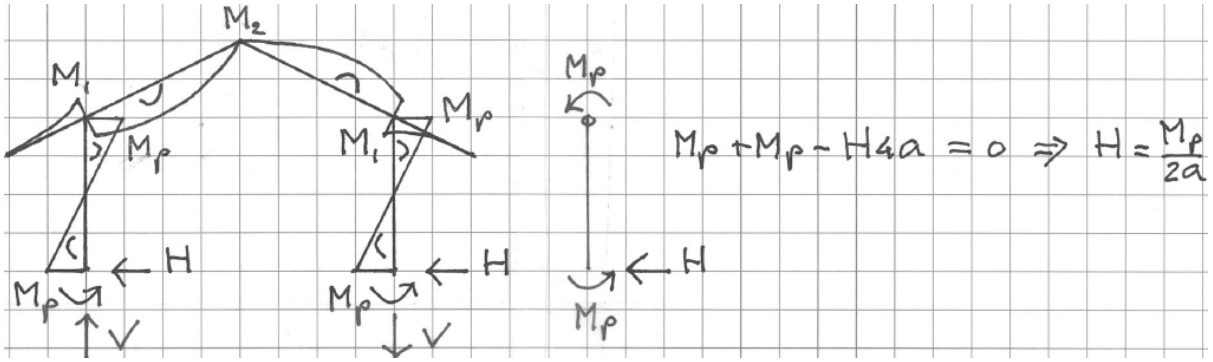


$$E = M_p\vartheta + M_p(\vartheta+\vartheta) + 2M_p(\vartheta+\vartheta) + M_p\vartheta$$

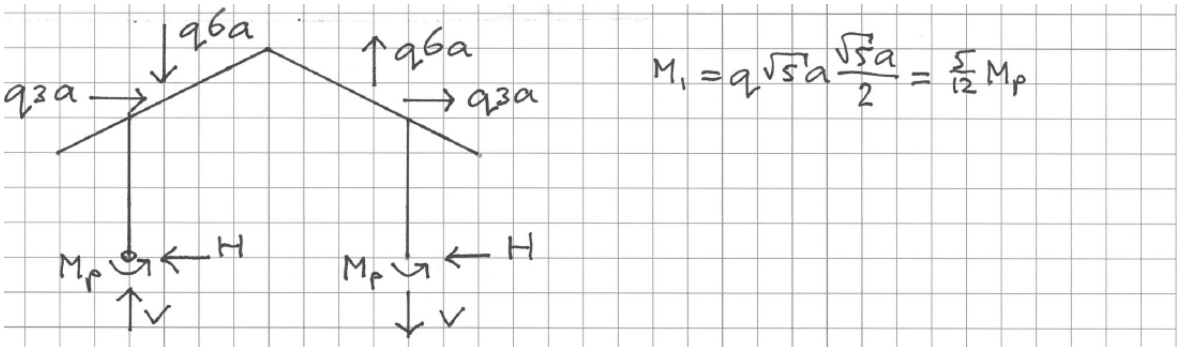
$$A = q3a\,\vartheta\tfrac{7}{2}a - q6a\,\vartheta a + q6a\,\vartheta a + q3a\,\vartheta\tfrac{1}{2}a$$

$$E = A \Rightarrow q = \frac{2}{3}\frac{M_p}{a^2}$$

Answer to problem 1b

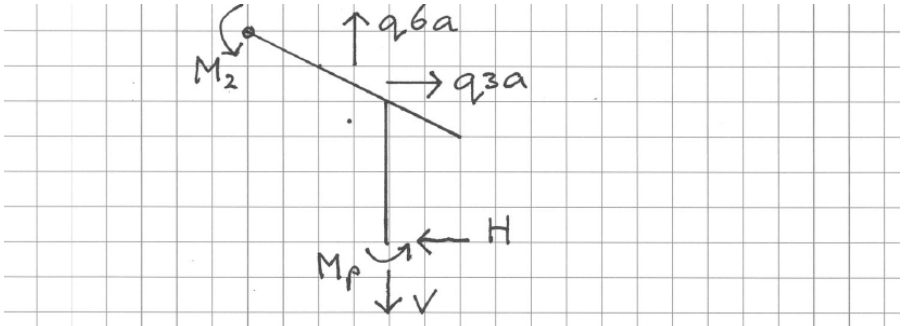


$$M_p + M_p - H4a = 0 \Rightarrow H = \frac{M_p}{2a}$$

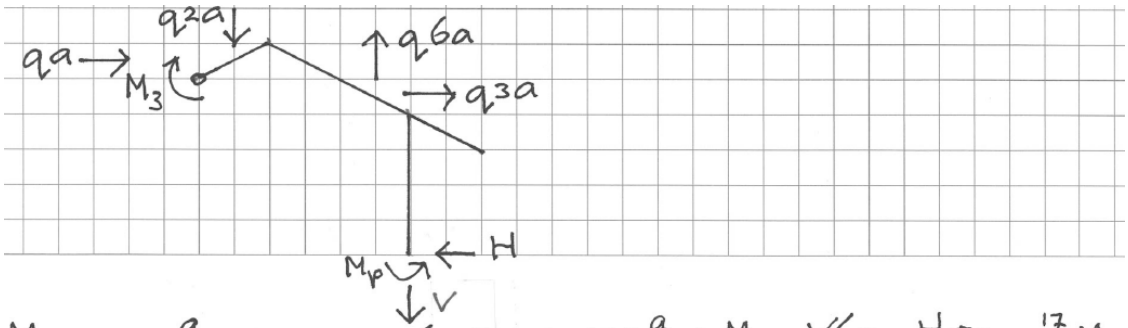


$$M_1 = q\sqrt{5}a\frac{\sqrt{5}a}{2} = \frac{5}{12}M_p$$

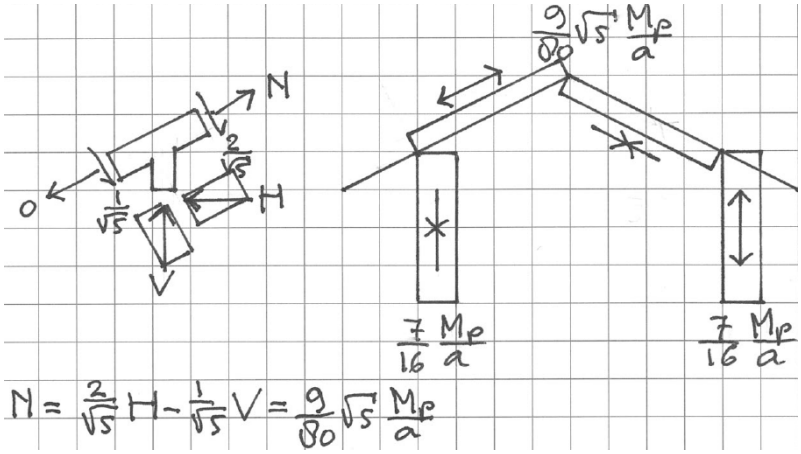
$$M_p = q3a\tfrac{9}{2}a + q6a\,a - q6a\tfrac{7}{2}a + q3a\tfrac{9}{2}a - M_p + V8a \Rightarrow V = \frac{7}{16}\frac{M_p}{a}$$



$$M_2 = -q6a\,3a - q3a\,\tfrac{3}{2}a + V4a + H6a - M_p = 0$$



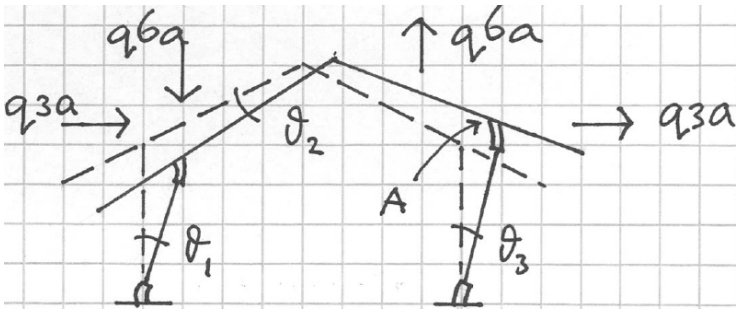
$$M_3 = -qa\tfrac{a}{2} - q2a\,a + q6a\,5a + q3a\tfrac{a}{2} + M_p - V6a - H5a = \tfrac{17}{24}M_p$$



$$N = \tfrac{2}{\sqrt{5}}H - \tfrac{1}{\sqrt{5}}V = \tfrac{9}{80}\sqrt{5}\,\tfrac{M_p}{a}$$

Answer to problem 1c

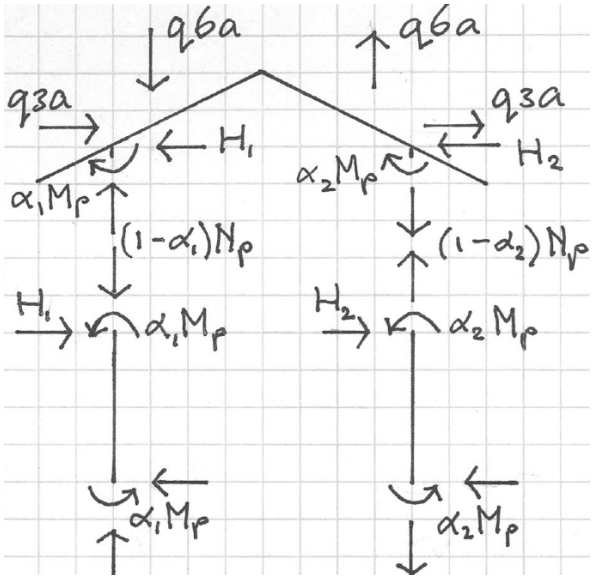
Upperbound



```
> b := 14 :
> # horizontal displacement of A
> eq1 := t1·4·a = t3·4·a :
> # vertical displacement of A
> eq2 := -t1·a/b - (t1 + t2)·a/b + t2·8·a = t3·a/b + (t2 + t3)·a/b :
> E := Mp·t1 + Mp·(t1 + t2) + Mp·(t2 + t3) + Mp·t3 :
> A := q·3·a·(t1·4·a - t2·a/2)·2 + q·6·a·(t1·a/b + (t1 + t2)·a/b - t2·a) + q·6·a·(-t1·a/b - (t1 + t2)·a/b + t2·7·a) :
> solve({eq1, eq2, E = A}, {t1, t2, q})
```

$$\left\{q = \frac{16\,Mp}{99\,a^2},\ t1 = t3,\ t2 = \frac{2\,t3}{55}\right\}$$

Lowerbound




```
> b := 14 : Np := b·Mp/a :
> eq1 := a1·Mp + a1·Mp = H1·4·a :
> eq2 := q·3·a - H1 + q·3·a - H2 = 0 :
> eq3 := q·6·a - (1 - a1)·Np - q·6·a + (1 - a2)·Np = 0 :
> eq4 := a1·Mp + q·3·a·a/2 + q·6·a·a - q·6·a·7·a + q·3·a·a/2 + a2·Mp + (1 - a2)·Np·8·a = 0 :
> eq5 := a2·Mp + a2·Mp = H2·4·a :
> solve({eq1, eq2, eq3, eq4, eq5}, {q, a1, a2, H1, H2});
```

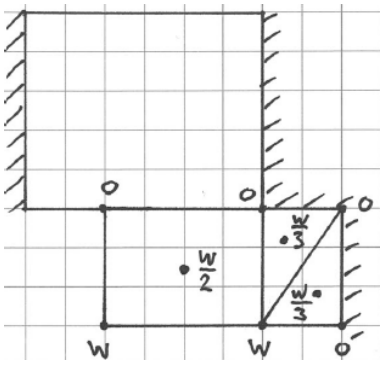
$$\left\{H1 = \frac{16\,Mp}{33\,a},\ H2 = \frac{16\,Mp}{33\,a},\ a1 = \frac{32}{33},\ a2 = \frac{32}{33},\ q = \frac{16\,Mp}{99\,a^2}\right\}$$

Answer to problem 2a

A, B, D, F

4 or less correct ..... 0.0 point
5 correct ..... 0.2 point
6 correct ..... 0.5 point
7 correct ..... 0.8 point
8 correct ..... 1.0 point

Answer to problem 2b

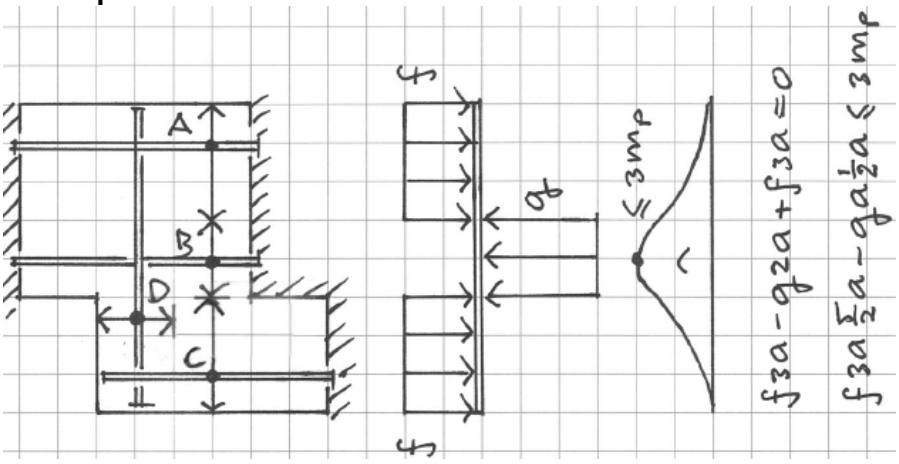


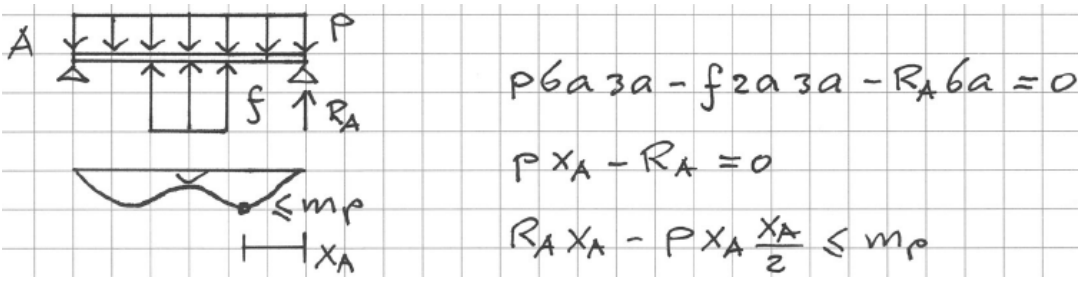
$$A = p\,3a\,4a\,\frac{w}{2} + p\,\frac{2a\,3a}{2}\,\frac{w}{3} + p\,\frac{2a\,3a}{2}\,\frac{w}{3} = 8\,p\,a^2 w$$

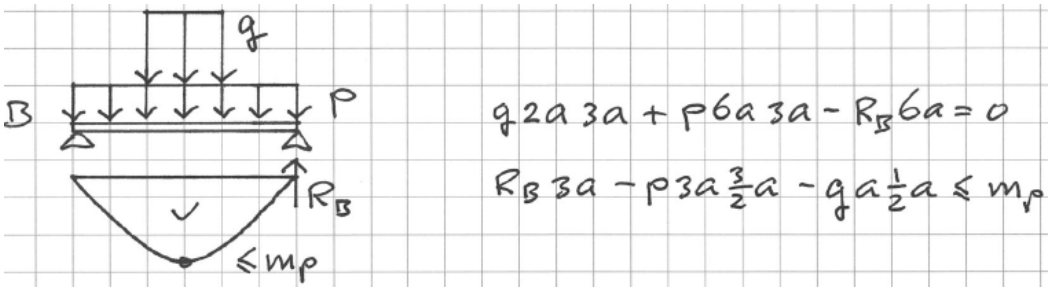
$$E = 3m_p\,\frac{w}{3a}\,4a + m_p\,2a\,\frac{w}{3a} + m_p\,3a\,\frac{w}{2a} = \frac{37}{6}\,m_p\,w$$

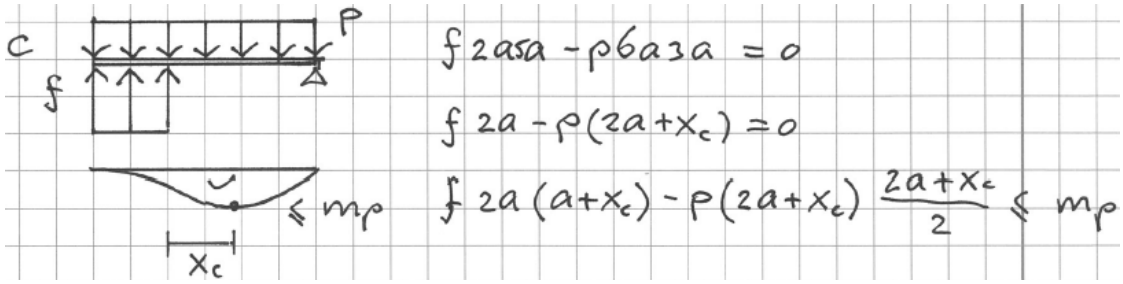
$$E = A \Rightarrow \quad p = \frac{37}{48}\,\frac{m_p}{a^2}$$

Answer to problem 2c










```
> eq1 := f·3·a − g·2·a + f·3·a = 0 :
> eq2 := f·3·a·5/2·a − g·a·1/2·a = 3·mp :
> eq3 := p·6·a·3·a − f·2·a·3·a − RA·6·a = 0 :
> eq4 := p·xA − RA = 0 :
> eq5 := RA·xA − p·xA·xA/2 = mp :
> eq6 := g·2·a·3·a + p·6·a·3·a − RB·6·a = 0 :
> eq7 := RB·3·a − p·3·a·3/2·a − g·a·1/2·a = mp :
> eq8 := f·2·a·5·a − p·6·a·3·a = 0 :
> eq9 := f·2·a − p·(2·a + xC) = 0 :
> eq10 := f·2·a·( a + xC) − p·(2·a + xC)·(2·a + xC)/2 = mp :
> opl := solve({eq1, eq3, eq4, eq6, eq7, eq8, eq9}, {p, f, g, RA, xA, RB, xC}); assign(opl) :
```

$$opl := \left\{RA = \frac{mp}{15\,a},\ RB = \frac{7\,mp}{15\,a},\ f = \frac{mp}{10\,a^2},\ g = \frac{3\,mp}{10\,a^2},\ p = \frac{mp}{18\,a^2},\ xA = \frac{6\,a}{5},\ xC = \frac{8\,a}{5}\right\}$$

```
> simplify(eq2);
```

$$\frac{3\,mp}{5} = 3\,mp$$

```
> simplify(eq5);
```

$$\frac{mp}{25} = mp$$

```
> simplify(eq7);
```

$$mp = mp$$

```
> simplify(eq10);
```

$$\frac{4\,mp}{25} = mp$$

Answer to problem 3

a Tresca

b - upperbound, mechanism, virtual work equation
- lowerbound, equilibrium, safe

c D