Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 16 June 2022, 13:30 – 16:30 hours

Write your <u>name</u> and <u>study number</u> at the top of your work.

Also write whether you were a <u>member</u> of the elastic team, plastic team or no team.

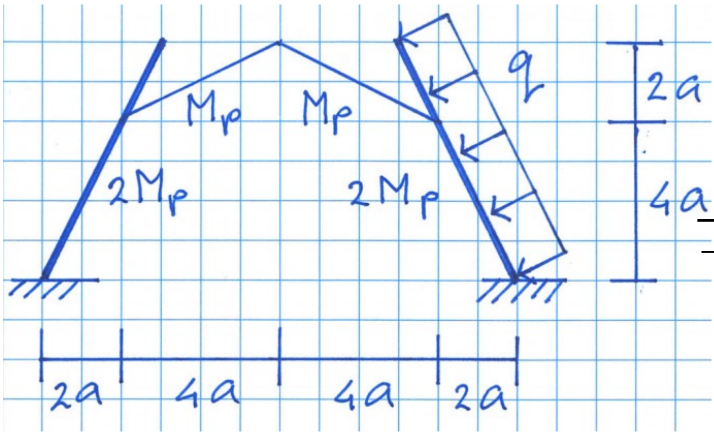


Figure 1. Frame structure

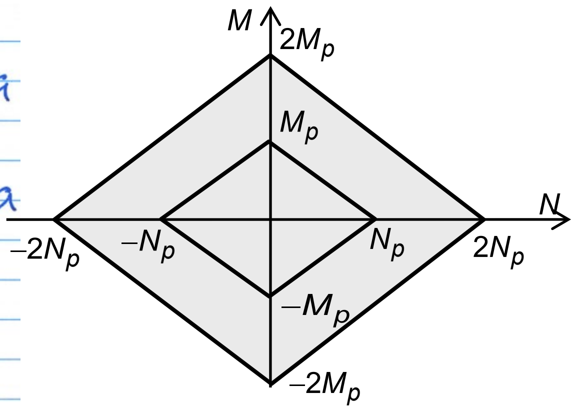


Figure 2. Yield contours

Problem 1

A frame consists of four members (Fig.1). The columns have a strength $2M_p$. The roof members have a strength M_p. The members are rigidly connected. The supports are fixed. The structure is loaded by an evenly distributed line load q per column length (wind load). The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 6$. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for q.
– Determine the smallest <u>upper-bound</u> for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has simply supported edges and free edges (Fig. 3). It carries an evenly distributed load *p* [kN/m²]. There is no other load on the plate. The plate is homogeneous and orthotropic.

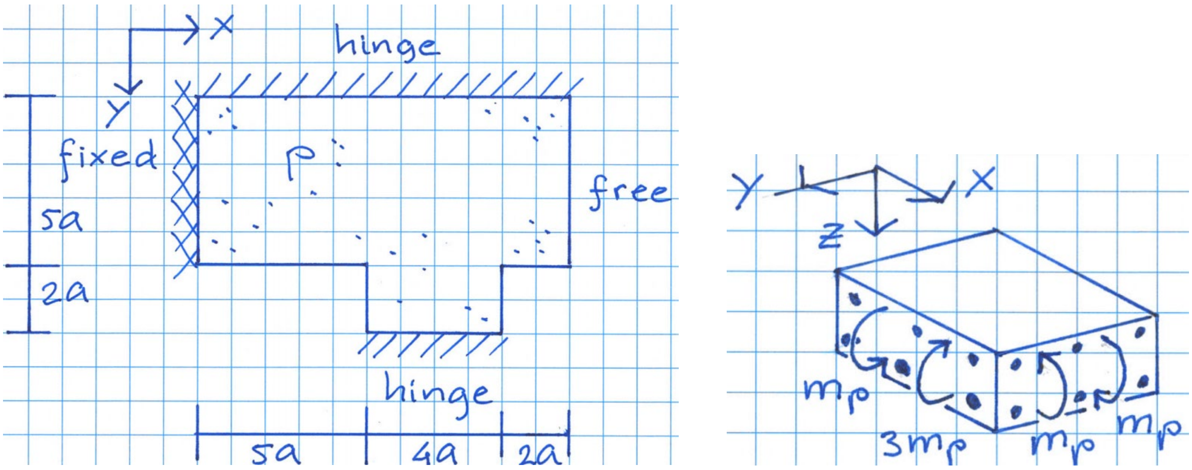


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

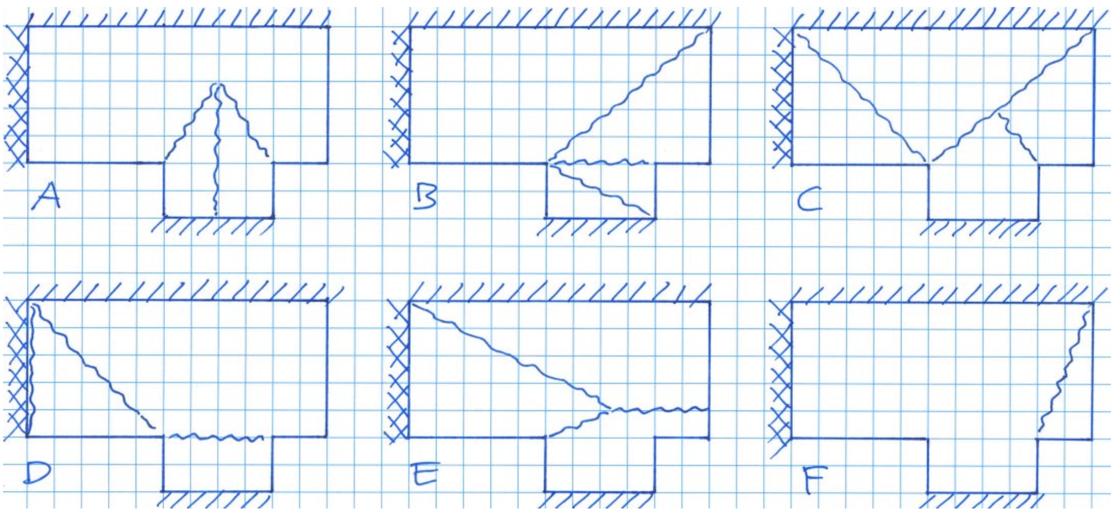


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper-bound</u> for p expressed in m_p and a (1.5 point).

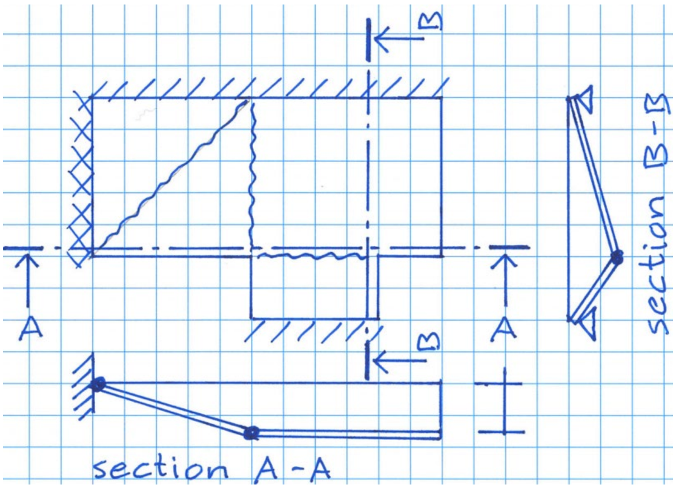


Figure 5. Mechanism of Problem 2b

c Determine the largest <u>lower-bound</u> for p using torsion free beams ($m_{xy} = 0$). You only need to write down the equations and not solve the equations. (1.5 point)

Problem 3

a What is ductility? Choose A, B, C or D. (0.5 point)

A The ability to deform with much plastic deformation.
B Plastic deformation in the direction of the normal vector on the yield contour.
C The material property that the complementary energy remains positive semi-definite.
D Shifting of the yield contour during strengthening of yielding.

b Welding stresses and rolling stresses reduce the … Choose A, B, C or D. (0.5 point)

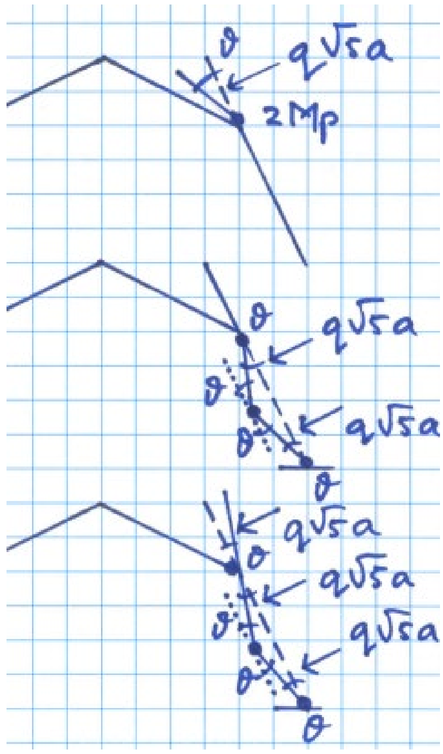
A … moment capacity of a beam cross-section.
B … buckling load of a column.
C … deflection of a beam.
D … interaction of plastic moment en normal force in a column cross-section.

c The yield lines in a plate transfer bending moments and torsion moments.
The torsion moments are neglected in calculating the virtual work.
Why? Choose A, B, C or D. (0.5 point)

A The stresses due to the torsion moments are very small.
B The torsion moments must be zero because the material has no extra capacity.
C A rhombic interaction diagram is assumed.
D There is no torsion deformation in a yield line.

Answer to problem 1a

```
> l:= sqrt(a^2+(2*a)^2):
> E:= 2*Mp*t:
> A:= q*l * 1/2 * t:
> q:= solve(E=A,q); evalf(q);
```



$$q := \frac{4\,Mp}{5\,a^2}$$
$$\frac{0.8000000000\,Mp}{a^2}$$

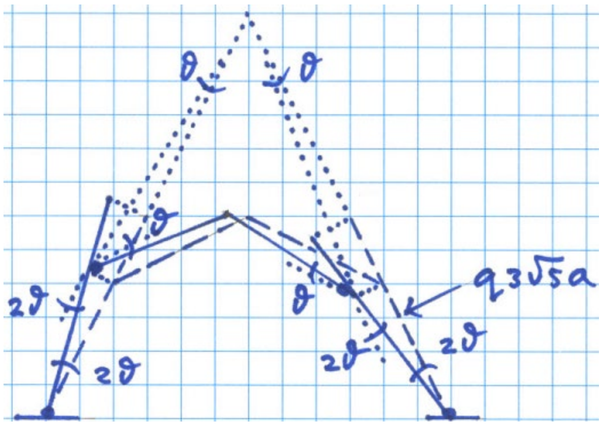
```
> E:= 2*Mp*t + 2*Mp*(t+t) + 2*Mp*t:
> A:= 2*( q*l * 1/2 * t):
> q:= solve(E=A,q); evalf(q);
```

$$q := \frac{8\,Mp}{5\,a^2}$$
$$\frac{1.600000000\,Mp}{a^2}$$

```
> E:= Mp*t + 2*Mp*(t+t) + 2*Mp*t:
> A:= q*l*1/2*t + q*l*1/2*t - q*l*1/2*t:
> q:= solve(E=A,q); evalf(q);
```

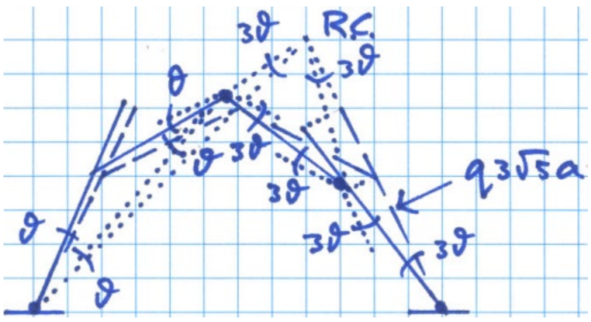
$$q := \frac{14\,Mp}{5\,a^2}$$
$$\frac{2.800000000\,Mp}{a^2}$$

```
> E:= 2*Mp*2*t + Mp*(2*t+t) + Mp*(2*t+t) + 2*Mp*2*t:
> A:= q*3*l * 3/2*l *2*t:
> q:= solve(E=A,q); evalf(q);
```



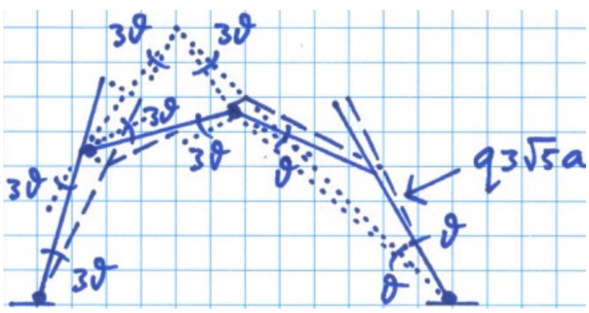
$$q := \frac{14\,Mp}{45\,a^2}$$
$$\frac{0.3111111111\,Mp}{a^2}$$

```
> E:= 2*Mp*t + Mp*(t+3*t) + Mp*(3*t+3*t) + 2*Mp*3*t:
> A:= q*3*l * 3/2*l * 3*t:
> q:= solve(E=A,q); evalf(q);
```



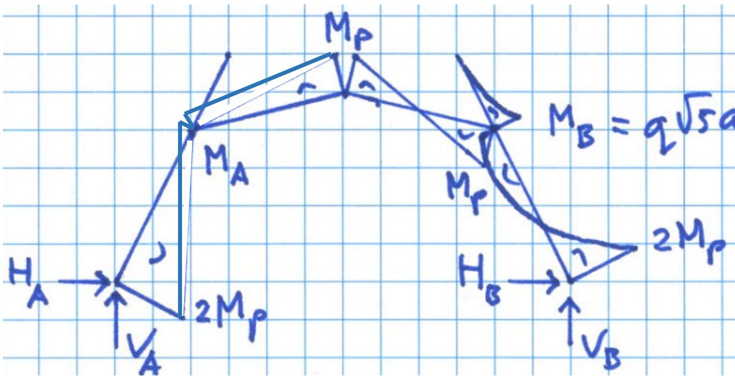
decisive $\boxed{q := \frac{4\,Mp}{15\,a\sim^2}}$
$$\frac{0.2666666667\,Mp}{a\sim^2}$$

```
> E:= 2*Mp*3*t + Mp*(3*t+3*t) + Mp*(3*t+t) + 2*Mp*t:
> A:= q*3*l * 3/2*l * t:
> q:= solve(E=A,q); evalf(q);
```

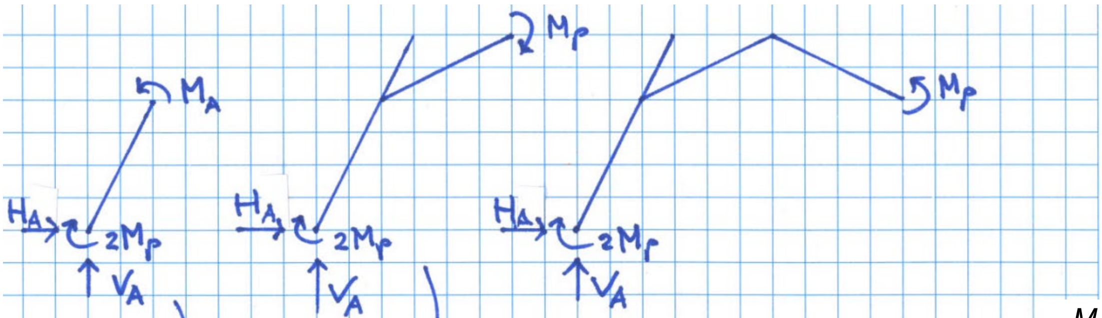


$$q := \frac{4\,Mp}{5\,a\sim^2}$$
$$\frac{0.8000000000\,Mp}{a\sim^2}$$

Answer to problem 1b



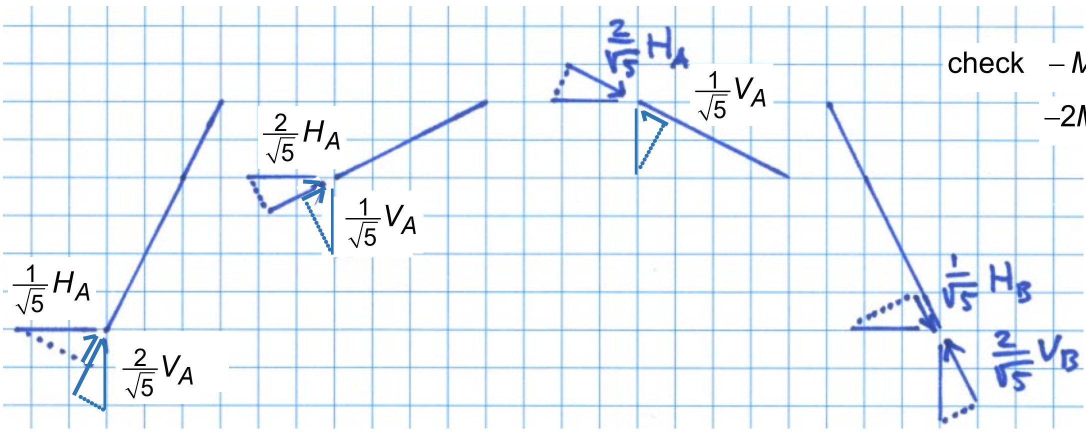
$$M_B = q\sqrt{5}a\,\frac{\sqrt{5}a}{2} = \frac{2}{3}M_p$$



$$\left.\begin{aligned} M_p &= V_A 10a - H_A 4a + 2M_p \\ M_p &= H_A 6a - V_A 6a - 2M_p \end{aligned}\right\} \quad H_A = \frac{2}{3}\frac{M_p}{a}, \quad V_A = \frac{1}{6}\frac{M_p}{a}$$

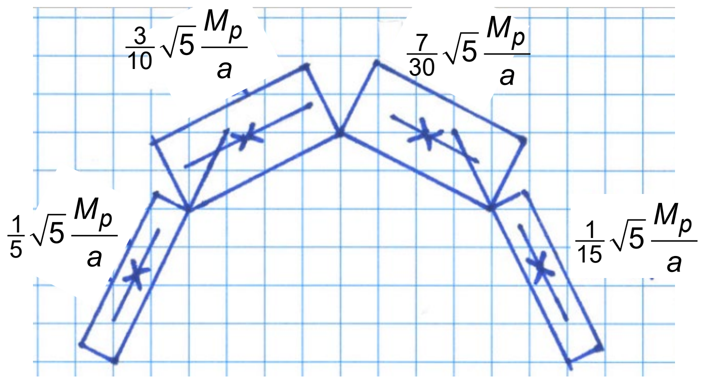
$$M_A = V_A 2a - H_A 4a + 2M_p = -\frac{1}{3}M_p$$

$$\left.\begin{aligned} H_A + H_B &= q6a \\ V_A + V_B &= q3a \end{aligned}\right\} \quad H_B = \frac{14}{15}\frac{M_p}{a}, \quad V_B = \frac{19}{30}\frac{M_p}{a}$$



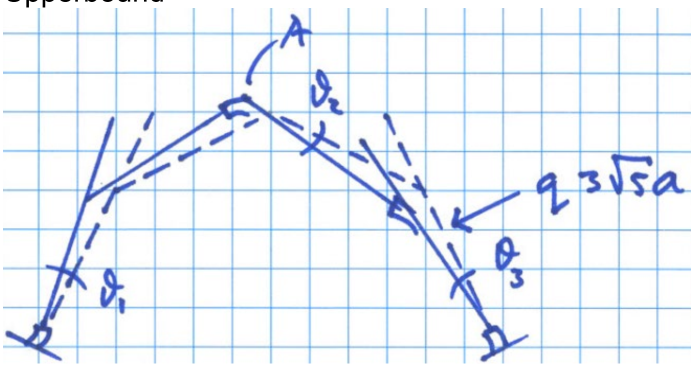
check $\quad -M_p \le M_A \le M_p$

$-2M_p \le M_B \le 2M_p \quad$ okay



Answer to problem 1c

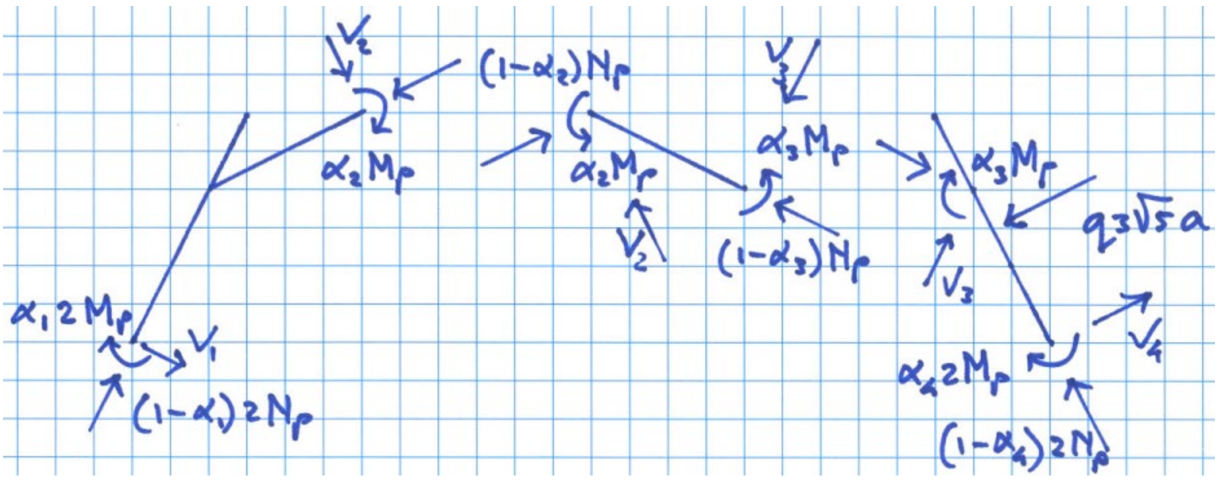
Upperbound




```
> b:=6:
> A_hor_left:=  t1*6*a + t1*a/b*1/sqrt(5) + (t1+t2)*a/b*2/sqrt(5):
> A_vert_left:= t1*6*a - t1*a/b*2/sqrt(5) - (t1+t2)*a/b*1/sqrt(5):
> A_hor_right:=   t3*4*a - t3*a/b*1/sqrt(5) - t2*2*a - (t2+t3)*a/b*2/sqrt(5):
> A_vert_right:= -t3*2*a - t3*a/b*2/sqrt(5) + t2*4*a - (t2+t3)*a/b*1/sqrt(5):
> E:= 2*Mp*t1 + Mp*(t1+t2) + Mp*(t2+t3) + 2*Mp*t3:
> A:= q*3*sqrt(5.)*a * 3*sqrt(5)*a/2 * t3:
> solve({A_hor_left=A_hor_right,A_vert_left=A_vert_right,E=A},{q,t2,t3});
```

$$\left\{q = \frac{0.2512774835\,Mp}{a^2},\ t2 = 3.566916657\,t1,\ t3 = 3.818693770\,t1\right\}$$

Lowerbound



```
> b:=6: Np:=b*Mp/a:
> eq1:= V1*2/sqrt(5.) + (1-a1)*2*Np*1/sqrt(5) + V2*1/sqrt(5) - (1-a2)*Np*2/sqrt(5) =0:
> eq2:= V1*1/sqrt(5 ) - (1-a1)*2*Np*2/sqrt(5) + V2*2/sqrt(5) + (1-a2)*Np*1/sqrt(5) =0:
> eq3:= a1*2*Mp = (1-a2)*Np*2/sqrt(5)*6*a - (1-a2)*Np*1/sqrt(5)*6*a - V2*1/sqrt(5)*6*a - V2*
  2/sqrt(5)*6*a - a2*Mp:
>
> eq4:= (1-a2)*Np*2/sqrt(5) - V2*1/sqrt(5) - (1-a3)*Np*2/sqrt(5) - V3*1/sqrt(5) =0:
> eq5:=-(1-a2)*Np*1/sqrt(5) - V2*2/sqrt(5) - (1-a3)*Np*1/sqrt(5) + V3*2/sqrt(5) =0:
> eq6:= a2*Mp = V3*2*sqrt(5)*a - a3*Mp:
>
> eq7:= (1-a3)*Np*2/sqrt(5) + V3*1/sqrt(5) - (1-a4)*2*Np*1/sqrt(5) + V4*2/sqrt(5) - q*6*a =0:
> eq8:= (1-a3)*Np*1/sqrt(5) - V3*2/sqrt(5) - (1-a4)*2*Np*2/sqrt(5) - V4*1/sqrt(5) + q*3*a =0:
> eq9:= a3*Mp = V4*2*sqrt(5)*a - q*3*sqrt(5)*a*1/2*sqrt(5)*a - a4*2*Mp:
>
> solve({eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8,eq9},{q,a1,a2,a3,a4,V1,V2,V3,V4});
```

$$\left\{V1 = \frac{0.4868551869\, Mp}{a},\ V2 = -\frac{0.1458383667\, Mp}{a},\ V3 = \frac{0.4060763047\, Mp}{a},\ V4 = \frac{1.068646448\, Mp}{a},\ a1 = 0.9661603079,\ a2 = 0.8971709740,\ a3 = 0.9188574688,\ a4 = 0.9878468028,\ q = \frac{0.2512774836\, Mp}{a^2}\right\}$$

Answer to problem 2a

B, D, F

3 or less correct …….	0.0 point
4 correct …………….	0.3 point
5 correct …………….	0.7 point
6 correct ……………..	1.0 point

Answer to problem 2b

```
> E:= p*5*a*5*a/2*w/3 + p*5*a*6*a*w/2 + p*2*a*4*a*w/2;
```

$$E := \frac{139}{6} p\, a^2\, w$$

```
> A:= mp*5*a*w/(5*a) + mp*5*a*w/(5*a) + mp*5*a*w/(5*a) + 3*mp*4*a*(w/(2*a)+w/(5*a));
```

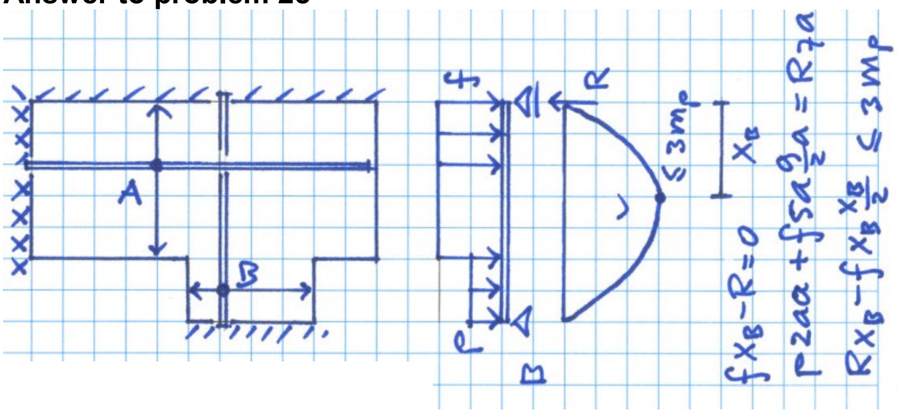
$$A := \frac{57}{5} mp\, w$$

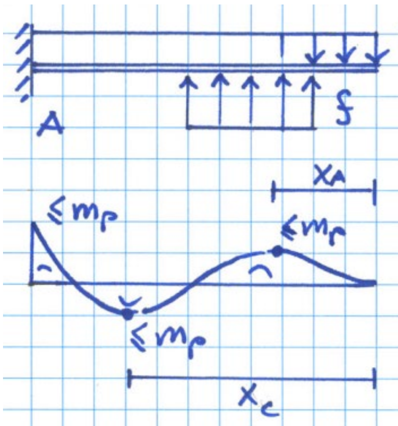
```
> p:=solve(E=A,p); evalf(p);
```

$$p := \frac{342}{695} \frac{mp}{a^2}$$

$$\frac{0.4920863309\, mp}{a^2}$$

Answer to problem 2c





$$p x_A - f(x_A - 2a) = 0$$

$$p x_A \frac{x_A}{2} - f(x_A - 2a)\frac{x_A - 2a}{2} \leq m_p$$

$$p x_c - f 4a = 0$$

$$f 4a (x_c - 4a) - p x_c \frac{x_c}{2} \leq m_p$$

$$p\, 11a \frac{11a}{2} - f 4a\, 7a \leq m_p$$

```
> eq1:= f*xB-R=0:
> eq2:= p*2.*a*a+f*5*a*9/2*a=R*7*a:
> eq3:= R*xB-f*xB*xB/2=3*mp:
> eq4:= p*xA-f*(xA-2*a)=0:
> eq5:= p*xA*xA/2-f*(xA-2*a)*(xA-2*a)/2=mp:
> eq6:= p*xC-f*4*a=0:
> eq7:= f*4*a*(xC-4*a)-p*xC*xC/2<mp:
> eq8:= p*11*a*11*a/2-f*4*a*7*a<mp:
> opl:=solve({eq1,eq2,eq3,eq4,eq5,eq6},{R,xA,xB,xC,p,f}); assign(opl):
```

$$opl := \left\{R = \frac{1.789727914\, mp}{a}, f = \frac{0.5338543344\, mp}{a^2}, \boxed{p = \frac{0.2581864372\, mp}{a^2}}, xA = 3.873170143\, a, xB = 3.352464893\, a, xC = 8.270834675\, a\right\}$$

```
> evalf(eq7);
```

$$0.289172532\, mp < mp$$

```
> evalf(eq8);
```

$$0.67235809\, mp < mp$$

Answer to problem 3

a A
b B
c B (D is not correct for yield lines that are inclined to the reinforcement directions: the bending rotation in one direction is the torsion rotation in the other.)